New Developments in Aggregation Economics

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Abstract

The goal of this article is to provide a general characterization of the aggregate behavior of a group in a market environment. We allow for public and private consumption, intragroup production, and consumption externalities within a group; we only assume that the group always reaches Pareto-efficient decisions. We show that aggregation problems involve a simple mathematical structure: The aggregate demand of the group, considered as a vector field, can be decomposed into a sum of gradients. We briefly introduce exterior differential calculus as a tool to study this structure. We analyze two main issues. One is testability: What restrictions (if any) on the aggregate demand function characterize the efficient behavior of the group? The second issue relates to identifiability; we investigate the conditions under which it is possible to recover the underlying structure—namely, individual preferences, the decision process, and the resulting intragroup transfers—from the group's aggregate behavior.

1. INTRODUCTION

The notion of aggregation is pervasive in economics.[1] Many (arguably most) economic decisions are made by groups, not individuals, with firms as an obvious example. It has long been recognized that the standard model of a unique, profit-maximizing decision unit often must be extended to take into account the multiperson nature of the decision process. The same remark applies to committees, clubs, villages, and other local organizations, which have also attracted much interest. Even standard micro demand analysis, although it routinely uses the tools of consumer theory, exploits data on households or families, which in general gather several individuals. Partial equilibrium analysis relies on aggregate demand or supply functions. And, quite obviously, macroeconomics concentrates on the aggregate behavior of vast classes of agents (households, firms, etc.), each being routinely identified with a single decision maker.

In all these cases, aggregation issues are raised, at least implicitly. When can a multiperson entity be analyzed as a single decision maker (i.e., when is there a representative consumer)? What data are needed to fully summarize the situation of a group? Are there testable restrictions on aggregate behavior stemming from the utility- (or profit-) maximizing actions of each member? Can one formulate welfare evaluations at the aggregate level, and what are their implications for the individuals under consideration? To what extent is it possible to recover information on individual-level characteristics (e.g., preferences, resources) or the intragroup decision process from the sole observation of aggregate behavior?

Quite often, answers to these questions are taken for granted without much analysis; for instance, macro models typically assume the existence of a representative agent with little discussion of either the prerequisites for the assumption or its implications. Still, theoretical investigations of the aggregation issues just mentioned, and of many others, have been available for several decades. Moreover, the field of aggregation theory has recently attracted renewed interest. Old, open problems have been solved; existing questions have been reconsidered from a different perspective; and more generally, a new subfield has emerged, with original emphasis, techniques, and results. In this review, we survey some of these recent developments.

The structure of the review is as follows. We first describe the notations we use throughout the article. Section 3 then briefly summarizes the main features of traditional aggregation theory, as it has developed up to the late 1970s and early 1980s. Section 4 describes how some of the traditional questions have recently been solved or reinterpreted. The recent literature on the aggregate behavior of small groups (i.e., aggregation in the small) is covered in Section 5.

Finally, one caveat is in order. The goal of this article is simply to provide a quick overview of some recent results. For the sake of brevity, we omit the proofs of some of the most important (and most complex) results, as well as many interesting but specific developments. The interested reader is referred to our recent book (Chiappori & Ekeland 2009a) for a more complete exposition.

[1] Throughout the article, we consider issues related to aggregation over individuals. The word aggregation is sometimes used in a totally different context—namely, the aggregation of several commodities into some composite good (e.g., the aggregation of various types of meat, vegetables, and dairy into the general category of food). The aggregation of commodities is not considered in this article.

2. NOTATION

We first define the notation used throughout the article. In what follows, the transpose of vector x is denoted x^T, and the scalar product of vectors x and y is denoted $x^T y$.

2.1. Commodities

We consider a group consisting of H members. Agents may consume M commodities, I of which are privately consumed by each member, while the remaining $J = M - I$ are public within the group; formally, the list of commodities may include leisure. Moreover, a given, physical commodity may be further indexed by the period or the state of the world (or both) at which it is available. Therefore, our setting extends to intertemporal behavior (savings, investment, human capital accumulation) as well as risk sharing and group decision under uncertainty.

Let x_h^i denote the private consumption of commodity i by group member h, and X^j the group's consumption of public good j. An allocation is a $J + HI$ vector $(X, x_1, \ldots, x_H)$, where

$$X = \left(X^1, \ldots, X^J\right) \in \mathbb{R}^J$$

and

$$x_h = \left(x_h^1, \ldots, x_h^I\right) \in \mathbb{R}^I \text{ for } h = 1, \ldots, H,$$

and the group's aggregate demand is the vector $(X, x) \in \mathbb{R}^M$, where $x = \sum_h x_h$. For brevity, the vector (X, x) is often denoted ξ.

2.2. Utility Functions

We assume that each person has a utility function over allocations. We denote h's utility function by $U^h(X, x_1, \ldots, x_H)$. This formulation is fully general; it allows the utility of h to depend on the private consumption of other members in a nonrestricted way. This interaction may be the result of altruism (i.e., h cares about other members' well-being) or paternalism (h is concerned with her partners' consumptions); it may also reflect other external impacts between consumptions (e.g., a member's smoking bothers the other members by reducing their utility, an intragroup externality in the usual sense). In particular, other members' consumption of private goods may impact h's marginal rate of substitution between her own private and public goods; in other words, we do not impose separability restrictions so far.

The utility functions $U^h, h = 1, \ldots, N$, are assumed continuously differentiable and strictly concave. In some cases, one will require stronger restrictions (e.g., infinite differentiability; strong concavity, requiring that the matrix of second derivatives is negative definite everywhere; or strong quasi-concavity, requiring that the restriction of this matrix to the subspace orthogonal to the gradient is negative definite).

Although quite reasonable, the form just described is sometimes too general—if only because it is difficult to incorporate such preferences into a model in which agents live alone for some part of their life cycle. Consequently, in many models preferences are egoistic, of the form $U^h(X, x_h)$. Finally, a fraction of the literature deals with market economies. In this context, preferences are strictly egoistic, and all commodities are privately consumed. In particular, interactions between group members (if any) are restricted to commodity trading. Then the general form just defined boils down to

$$U^h(X, x_1, \ldots, x_H) = u^h(x_h). \tag{1}$$

2.3. Aggregate Budget Constraint

Let p denote the price vector of private goods, P the price vector of public goods, and y the group's total income. Again, for brevity, the vector (P, p) is often denoted π, so the aggregate demand (as a function of prices and income) becomes $\xi(\pi, y)$.

The group has limited resources. Specifically, its purchase vector $\xi = (X, x)$ must satisfy a standard market budget constraint of the form

$$\pi^T \xi = P^T X + p^T \left(\sum_h x_h \right) \leq y.$$

Throughout, we assume that behavior is zero homogeneous in prices and income. For some computations, we therefore may normalize the group's total income to be one. Also, we sometimes consider the group's budget shares, defined by

$$\Psi = (\Psi_1, \ldots, \Psi_M) \text{ where } \Psi^i = \frac{\pi_i \xi^i}{y}.$$

3. STANDARD AGGREGATION THEORY: A BRIEF OVERVIEW

As a first step, it is useful to briefly reconsider some crucial aspects of aggregation theory as it has developed up until the early 1980s. Our goal here is not to provide a survey; the interested reader is referred to, for instance, Deaton & Muelbauer (1980) and Shafer & Sonnenschein (1982) for that purpose. Instead, we want to briefly recall the main features of this literature, and in particular the questions it asked and the answers it provided. The general notion of aggregation theory gathers a host of different and more or less related approaches. To provide an overview, we find it convenient to distinguish between two core approaches: one in which the group is considered as a (mostly exchange) economy and a more general perspective that allows for richer interactions such as public consumptions or intragroup production.

3.1. Groups as Market Economies

In this first subsection, we assume that all commodities are privately consumed; we thus rule out public goods, as well as externalities of any type, and do not consider intragroup production. A few questions have played a crucial role in the development of the various branches of the field, and we organize our presentation around them.

3.1.1. When does a group behave as a single decision maker? A first question relates to the conditions under which the aggregate behavior of a group can validly be described using the tools of standard consumer theory. A first version, which has been known for at least a century [since Antonelli's (1971[1886]) pathbreaking result], is the following. Assume that some total income y is distributed between H agents, who each freely spend his share on several goods. When is it the case that the group's aggregate demand for each good can be expressed as a function of y alone (i.e., does not depend on how the income has been allocated within the group)? The answer is straightforward: For the property to be satisfied, it must be the case that transferring a dollar from one group member to another does not change total consumption—in other words, the marginal propensity to consume (MPC) each good must be the same for all agents, irrespective of their income. This can only happen under two conditions: (*a*) Each agent's MPC is independent of the agent's income, and (*b*) these constant MPCs are identical across agents. In other words, individual Engel curves must be linear or affine, and the coefficients of income must be common to all agents. When it is the case, one can readily show that the resulting, aggregate demand is compatible with utility maximization.

Clearly, this statement sounds like an impossibility result. Although a group may in theory behave like a single individual, the restrictions required in practice are quite stringent. Cross-section consumer expenditure data provide strong evidence against hypotheses (*a*) and (*b*), at least under the standard assumption that, among consumers with the same observable characteristics, preferences are distributed independently of income.

This negative conclusion has led to a reformulation of the question along a slightly different line, usually called exact nonlinear aggregation. Assume, again, that total income is distributed between the agents, with agent h receiving some amount y_h. Lau (1982) asks, when one can find some (possibly vector-valued) aggregate statistic $\tilde{y}$, dependent on the distribution of income within the group, such that aggregate demands can be expressed as functions of prices and $\tilde{y}$ only? The answer is positive when individual demands have the form

$$x_h^i(p, y_h) = \sum_{k=1}^{K} a_h^{k,i}(p) b_h^{k,i}(y_h). \tag{2}$$

Gorman (1981) proves that for each h and p, the matrix $a_h^{k,i}(p)$ has rank 3 or less. Moreover, the maximum rank can be reached only for specific functions $b_k^h(y_h)$ (see Lewbel 1991 on such rank restrictions). A specification widely used in empirical applications is the PIGLOG form, in which $K = 2$, $b_h^{1,i}(y_h) = y_h$, and

$$b_h^{2,i}(y_h) = y_h \ln(y_h).$$

This yields a so-called flexible functional form (Lau 1982), with individual budget shares of the form

$$\psi_h^i(p, y_h) = p_i a_h^{1,i}(p) + p_i a_h^{2,i}(p) \ln(y_h). \tag{3}$$

If, in addition, $a_h^{1,i}, a_h^{2,i}$, and b_h^i are independent of h, then aggregate demand becomes

$$\Psi^i = p_i a^{1,i}(p) + p_i a^{2,i}(p) \ln(\tilde{y}),$$

where $\tilde{y}$ is defined by

$$\ln \tilde{y} = \frac{\sum_h y_h \ln(y_h)}{\sum_h y_h},$$

which is closely related to Theil's inequality index.[2]

[2]Specifically, Theil's index is defined by $T_1 = \frac{\sum_h y_h \ln(y_h)}{\sum_h y_h} - \ln(\bar{y}) = \ln(\tilde{y}/\bar{y})$, where $\bar{y}$ denotes average income.

In that case, aggregate market shares Ψ_i only depend on individual incomes through the aggregate indicator $\tilde{y}$. Moreover, they have exactly the same form as the individual market shares in Equation 3; therefore, they can be derived from the maximization of a utility of the same form, using the aggregate indicator $\tilde{y}$ as the group's pseudo-income. It follows that, in terms of budget shares, the group behaves like a single agent, endowed with the same utility as each individual in the group and a pseudo-income equal to $\tilde{y}$. Moreover, as the same functional form can be used at both the individual and the aggregate levels, this specification may be useful when individual data do not display enough price variation. Indeed, a strong motivation for earlier research was the design of a demand function that would allow the estimation of Engel curves from cross-sectional data and of price effects from aggregate time series.

One can see from this example that one drawback of Antonelli's restrictions—the lack of realism of individual demands—is largely alleviated by the nonlinear aggregation approach. In fact, state-of-the-art estimations of individual demands routinely use functional forms such as the quadratic almost ideal demand system (Banks et al. 1997) with budget shares of the rank-3 Gorman form:

$$\psi_h^i(p, y_h) = p_i a_h^{1,i}(p) + p_i a_h^{2,i}(p) \ln(y_h) + p_i a_h^{3,i}(p) \ln^2(y_h).$$

However, the second problem remains: Aggregation is possible only under very strong restrictions regarding heterogeneity of preferences (in our example, all agents must have identical preferences, although this can be slightly relaxed). In short, a representative consumer may exist for acceptably general individual demands—but only when agents have similar preferences.[3]

A last remark is that one should be cautious with welfare judgments suggested by the representative consumer's utility. Indeed, several authors have provided examples showing that a given reform may increase the utility of the representative consumer while decreasing the welfare of all individuals in the economy (see, for instance, Jerison 1984b, Dow & da Costa Werlang 1988).

3.1.2. Is some structure preserved by (large-scale) aggregation? A second line of research adopts the opposite viewpoint. Instead of imposing a priori some desired structural properties of aggregate demand (e.g., the existence of a representative consumer) and trying to find sufficient conditions on individual preferences for these properties to be satisfied, these approaches consider general preferences and ask which structure (if any) aggregate demand must have, given that each individual in the economy maximizes a well-behaved utility under a budget constraint. We have known for a long time that utility maximization generates a lot of structure for individual demands (namely, the symmetry and negativeness of the Slutsky matrix); the question is whether (some of) this structure is preserved by aggregation.

The problem was initially raised by Sonnenschein (1973) in a seminal article. Technically, Sonnenschein states two versions of the problem. In both cases, individuals each maximize utility under a budget constraint, but in the first version each individual

[3]More heterogeneity of individual preferences can be allowed if the distribution of income is restricted, for example, if the consumers get fixed shares of aggregate income, as in the case of workers with different skill levels and no other capital (see Jerison 1984a,b; Mas-Colell et al. 1994, chapter 4).

h receives some nominal income y_h, whereas in the second each receives a fixed endowment ω_h. Technically, individual maximization programs are therefore of the form

$$\begin{gathered}\max_{x_h} U^h(x_h) \text{ subject to} \\ p^T x_h = y_h\end{gathered} \tag{4}$$

in the first case and

$$\begin{gathered}\max_{x_h} U^h(x_h) \text{ subject to} \\ p^T x_h = p^T \omega_h\end{gathered} \tag{5}$$

in the second. The first case corresponds to the market demand problem; the second determines the agent's excess demand, defined as the difference between the agent's desired consumption bundle and her initial endowment.

In both cases, aggregate demand depends on prices and initial endowments. The main question, however, is with regard the characterization of aggregate demand as a function of prices. It can therefore be stated as follows: Consider a given function $x(p)$. When is it possible find H smooth, increasing, strongly concave utility functions U^h and (*a*) (market demand) H scalars $(y_1, \ldots, y_H)$ such that

$$x(p) = \sum_h x_h(p),$$

where $x_h(p)$ solves Equation 4 and (*b*) (excess demand) H vectors $(\omega_1, \ldots, \omega_H)$ in $\mathbb{R}^N$ such that

$$x(p) = \sum_h x_h(p),$$

where $x_h(p)$ solves Equation 5.

The statement of the excess demand case can actually be slightly simplified. For any given set of direct utilities $\{U^1, \ldots, U^H\}$ and initial endowments $\{\omega_1, \ldots, \omega_H\}$, one can define the utilities $\{\tilde{U}^1, \ldots, \tilde{U}^H\}$ by $\tilde{U}^h(z) = U^h(z + \omega_h)$, $h = 1, \ldots, H$. With this notation, and defining $z_h = x_h - \omega_h$, the program in Equation 5 can be rewritten as

$$\begin{gathered}\max_{z_h} \tilde{U}^h(z_h) \text{ subject to} \\ p^T z_h \leq 0.\end{gathered} \tag{6}$$

There are obvious restrictions that the aggregate market or excess demand will satisfy. One is continuity (or differentiability in our context). Another is adding up (sometimes called the Walras law); namely, it must be the case that $p^T x(p) = \sum_h y^h$ for market demand, and $p^T z(p) = \sum_h p^T [x_h(p) - \omega_h] = 0$ for excess demand. Finally, excess demand functions are zero homogeneous in prices. The question is whether these properties are sufficient or whether the underlying structure generates stronger properties at the aggregate level.

In practice, a large fraction of the subsequent literature has been devoted to the case of a large economy, i.e., one in which the number of agents exceeds the number of commodities. Sonnenschein's conjecture was that in large economies, the obvious properties just listed fully characterize aggregate demands: Individual structure is therefore lost by aggregation, at least if the latter takes place on a sufficiently large scale.

Within months after Sonnenschein's initial statement, the excess demand case was independently solved by Mantel and Debreu [this literature is actually often referred to as Debreu-Mantel-Sonnenschein (DMS)]. Specifically, Mantel (1974) establishes that any smooth function satisfying homogeneity and adding up could be decomposed on any compact set of prices as the aggregate excess demand of an economy with at least $H=2M$ agents. Debreu (1974) shows that the result is valid for $H=M$, and Mantel (1976) proves that, in addition, one could assume that all utilities were homothetic.

Chiappori & Ekeland (2004) provide a short proof of a slightly stronger result. Their approach is based on the properties of individual excess demand functions. Surprisingly enough, given their theoretical importance, individual excess demands had not been studied in detail until recently. Their key result is the following. Suppose that $V(p)$ is a smooth function defined on some neighborhood $\mathcal{O}$ of $\bar{p}$, with $D_p V(\bar{p}) \neq 0$. Assume that it is quasi-convex, positively homogeneous of degree zero [which implies that $p^T \cdot D_p V(p) = 0$], that $D^2_{pp} V(p)$ has rank $(N-1)$, and that the restriction of $D^2_{pp} V(p)$ to $\mathrm{Span}\{p, D_p V(p)\}^{\perp}$ is positive definite. Take any C^2 function $\lambda(p) > 0$, homogeneous of degree (–1) on $\mathcal{O}$, and set

$$z(p) = -\frac{1}{\lambda(p)} D_p V(p) \tag{7}$$

so that $p^T z(p) = 0$. Then $z(p)$ is the excess demand function of some consumer; i.e., there exists a strictly quasi-concave function $U(z)$ defined and C^2 in a neighborhood $\mathcal{N}$ of $z(\bar{p})$, such that V is the indirect utility associated with U. It follows, in particular, that if $z(p)$ is an individual excess demand, then for any positive, zero-homogeneous scalar function $\zeta(p)$, $\zeta(p)z(p)$ is also an individual excess demand.

Consider now some compact subset K of the positive orthant. For $H \geq M$, take some family $V^h(p)$, $1 \leq h \leq H$, such that at every $p \in K$, the set of linear combinations of $DV^h(p)$ with nonnegative coefficients spans $T_p S^{N-1}$ (the tangent space $T_p S^{N-1}$ at p to the N-dimensional simplex, to which the price vector can be normalized to belong). Then for any C^2 map $x(p)$ defined on K, homogeneous of degree zero and satisfying the Walras law $p^T x(p) = 0$, one can find excess demand functions $z_h(p)$, $1 \leq h \leq H$, such that the decomposition $x(p) = \sum_h z_h(p)$ holds on K and the indirect utility associated with z_h is V^h. This version of the result is slightly stronger than Debreu's because the indirect utilities can be defined independently of the excess demand at stake; i.e., the same $V^1(p), \ldots, V^H(p)$ can be used in the decomposition of any given function.[4]

The DMS result was quite influential in the profession. Its theoretical implications are far from trivial; for instance, it immediately implies that for any compact subset of the positive orthant, one can always find economies with exactly one equilibrium within the compact subset, such that this equilibrium is not stable by the Walrasian tatonnement. From a more epistemological perspective, it has also been widely interpreted as a negative result: If aggregate demand can be anything, it was argued, then general equilibrium theory has no testable implication (except maybe the existence of an equilibrium), at least when applied to a large-enough economy. As shown below, this somewhat excessive claim, however, has been drastically reconsidered by the recent literature on the topic.

[4]This property may sound surprising. In sharp contrast with market demands, it reflects the existence of a continuum of individual excess demands that correspond to the same indirect utility (of course, they involve different initial endowments in general).

Finally, the case of a small economy (i.e., one with fewer agents than goods) has been considered by Diewert (1977) and Geanakoplos & Polemarchakis (1980). These authors show that, in such a setting, additional conditions (beyond the obvious ones) have to be fulfilled for a given function to be the aggregate excess demand of such an economy. Their approach relies on a local linearization of the problem; in particular, although the conditions they provide are indeed necessary, neither article provides sufficiency results. We come back to these results below.

3.1.3. Can aggregation create structure? The main conclusion of the Sonnenschein program is that assembling a sufficiently large number of sufficiently diverse utility maximizers may result in a collective demand with bizarre (in fact arbitrary) properties; in short, aggregation in the large tends to destroy any structure that may exist at the individual level (and in small groups; see below). An interesting question, however, relies on the opposite perspective: Is it the case that the aggregation of sufficiently diverse individual demands results in an object that is more regular that its component? In other words, can aggregation create structure?

This line of research has been pioneered by Hildenbrand. In a series of papers, Hildenbrand (1983, 1994) investigates the law of demand (LD) (see Hicks 1956). Denoting by $X(p)$ a demand function, it is said to satisfy the LD if the inequality

$$[X(p) - X(q)]^T(p - q) \leq 0$$

holds for all price systems p and q. If $X(p)$ is differentiable, it is equivalent to the Jacobian matrix

$$D_pX = \left(\frac{\partial X^i}{\partial p_j}\right)_{1\leq i,j\leq I}$$

being negative semidefinite at every p. Roughly speaking, the LD means that consumption and prices move in opposite directions. It implies that the demand curve for every good is downward sloping, but it is of course much more. For instance, if aggregate demand satisfies the LD, then the equilibrium is unique.

It is well known (e.g., Giffen goods) that individual demand functions need not satisfy the LD. For an individual having nominal income y, Marshallian demand is a function $x(p, y)$, and we have

$$D_px = S(p, y) - \left(D_yx\right)x',$$

where D_p and D_y denote partial derivatives. The first term on the right-hand side, $S(p, y)$, is the Slutsky matrix, which is negative definite. The second term on the right-hand side, $(D_yx)x'$, describes the income effect, that is, the change in wealth due to the change in prices. If preferences are homothetic, it is positive semidefinite, so the LD is satisfied. Apart from this special case, the income effect bites, and the LD needs not be satisfied at the individual level.

Hildenbrand's idea is that the LD can nevertheless be satisfied at the macroeconomic level because of special properties of the income distribution. This idea is best explained from the example in Hildenbrand (1983). Suppose all individuals have identical preferences, and the income distribution has a differentiable density $\mu(y)$ on $[0, \bar{y}]$ with $\mu(\bar{y}) = 0$. The aggregate demand is

$$X(p) = \int_0^{\bar{y}} x(p, y)\mu(y)dy.$$

The aggregate Slutsky matrix is obviously negative definite, and the aggregate income effect is

$$\sum \int_0^{\bar{y}} \frac{\partial x^i}{\partial y} x^i \mu(y) dy = \sum \int_0^{\bar{y}} \frac{\partial}{\partial y} \left(x^i\right)^2 \mu(y) dy$$
$$= -\sum \int \left(x^i\right)^2 \frac{d\mu}{dy} dy,$$

where we have integrated by parts. The two boundary terms vanish, the first one because $x^i(0) = 0$ and the second one because $\mu(\bar{y}) = 0$, and we are left with the integral term. Now a striking result emerges: If $\frac{d\mu}{\partial y} \leq 0$, that is, if the density is decreasing, then the aggregate income effect is positive definite, and the collective demand $X(p)$ satisfies the LD, even though the individual demands do not. Of course, it is unrealistic to assume that individuals have identical preferences, but this example vindicates the idea that particular properties of the wealth distribution can result in the LD. As shown in Chiappori (1985), one can also obtain the LD by putting conditions on the form of both individual demand and the wealth distribution, the Hildenbrand result, in which all preferences are identical, being just a polar case. The question is then to find characteristics of consumption and wealth distributions that (*a*) are empirically verifiable and (*b*) will generate the LD.

Jerison (1982, 1999) shows that aggregate demand satisfies the weak axiom of revealed preference (RP), a weak version of the LD, if there is increasing demand dispersion, that is, if the cloud of consumption vectors for individuals of a given income level is increasingly dispersed as the level rises. In other words, the Engel curves spread out at higher income levels. Grandmont (1992) decomposes a population into subclasses that are rescaled replicas of each other and shows that sufficient heterogeneity, as measured by the flatness of the density of the scale factors, leads to the LD. Kneip (1999) introduces a nonparametric notion of demand heterogeneity with the same result. Hildenbrand (1994) takes a different approach and checks directly, using British and French family expenditure data, that the aggregate income effect is positive definite (Härdle et al. 1991). There are a number of econometric problems to overcome. For instance, as such surveys do not follow an individual through time, one cannot infer from the data how a small change in income would affect the average consumption of individuals at a given income level. However, the surveys give the average consumption of individuals with slightly higher or slightly smaller incomes, and this should be a reasonable stand-in, provided the other characteristics of the population do not change dramatically across income classes. This being said, the econometric conclusions do seem to provide empirical support for the LD.

3.2. Groups as Complex Economies

A second line of research has considered aggregation problems from a totally different perspective. On the one hand, it essentially deals with small groups (typically households or families). On the other hand, instead of focusing on market economies, it considers potentially more complex interactions, involving possibly public consumption and intragroup production. The initial literature almost exclusively concentrates on one question—namely, which assumptions would guarantee that the group under consideration behaves like a single individual? We briefly describe the two main contributions to this literature: Samuelson's aggregate welfare index and Becker and Bergstrom's transferable utility (TU) setting.

3.2.1. Samuelson's index. Assume that all individuals agree on some global index, the arguments of which are the various individual utilities that will be maximized by the group. Technically, there exists some strictly increasing W such that the group maximizes

$$W\left[U^1(X, x_1, \ldots, x_H), \ldots, U^H(X, x_1, \ldots, x_H)\right] \tag{8}$$

under the budget constraint

$$P^T X + p^T\left(\sum_h x_h\right) = y. \tag{9}$$

Household production could be introduced at little cost; this task is left to the reader.

It is straightforward to see that this formulation boils down to a standard utility maximization problem. Indeed, define the group utility $\mathcal{U}^G$ by

$$\mathcal{U}^G(X, x) = \max_{\sum_h x_h = x} W\left[U^1(X, x_1, \ldots, x_H), \ldots, U^H(X, x_1, \ldots, x_H)\right].$$

Then $\mathcal{U}^G$ is the utility of a representative consumer for the group: for any consumption vector $(X, x_1, \ldots, x_H)$ that maximizes Equation 8 under Equation 9, the vector (X, x) where $x = \sum_h x_h$ maximizes $\mathcal{U}^G$ under the budget constraint $P^T X + P^T x = y$. Note that this conclusion is just a restatement of an old result by Hicks, sometimes referred to as the composite good theorem: In this setting, for any i the commodities $x_1^i, \ldots, x_H^i$ are always purchased at the same price p_i.

Simple as it may seem, this approach has some interesting properties. First, the relationship between $\mathcal{U}^G$, on the one hand, and $(W, U^1, \ldots, U^H)$, on the other hand, is not one-to-one: There exist many (in fact a continuum of different) structures $(W, U^1, \ldots, U^H)$ that generate the same representative utility U^G. This is exactly the spirit of Hick's theorem: Without variations in the respective prices of $(x_1, \ldots, x_H)$, individual utilities simply cannot be recovered. It follows that in this approach, the group is doomed to be a black box: Its aggregate behavior can certainly be studied (using standard consumer theory), but its inner mechanisms (individual utilities and the index W) are necessarily unrecoverable. Ironically, we see below that Samuelson's index is a particular case of a more general representation (the so-called collective approach), which only postulates that the group decisions are Pareto efficient. In this general family, simple exclusion restrictions are generically sufficient for individual utilities to be identified; the Samuelson index case is among the few exceptions for which individual utilities can never be recovered.

A second remark is that the Samuelson index case satisfies income pooling; that is, the group's behavior depends only on total income, not on its allocation between the group members. Therefore, in this setting, paying a benefit to one member instead of another (e.g., to the husband instead of the wife) cannot possibly have any impact on the outcomes. As shown below, there is strong empirical evidence against this prediction.

Finally, there is a relationship between the Samuelson index and the market economy approach described above, although the link is somewhat subtle. Assume for a moment that agents are egoistic and only consume private goods, and there are no externalities, so we are back in the setting studied in Section 2.1. Now the maximization of $W(U^1, \ldots, U^H)$ under a budget constraint generates a consumption plan that is Pareto efficient, for otherwise an alternative allocation would increase some of the U^h without decreasing any, but this would strictly increase W, a contradiction. By the second welfare theorem, this efficient allocation can be decentralized; i.e., there exists an income distribution within the group such that this allocation obtains as an equilibrium; in practice, if each agent h receives a specific income y_h (with $\sum_h y_h = y$) and consumes it at his will, the resulting consumption plan maximizes W under a budget constraint.[5] Clearly, this argument can be applied to any specific value of the price vector $p = (p_1, \ldots, p_M)$. The crucial remark, however, is that the income distribution that decentralizes the optimal allocation at prices p may (and generally will) depend on p in an arbitrary way. In particular, there is no reason to expect that it will be either constant, as in the market demand case, or a linear function of p, as in the excess demand case. In other words, in Samuelson's index story, there exists some income allocation $y(p) = [y^1(p), \ldots, y^H(p)]$ such that individuals behave as if they were maximizing their own utility under a budget constraint, solving a program of the form

$$\begin{array}{c} \max_{x_h} U^h(x_h) \quad \text{subject to} \\ p^T x_h = y^h(p). \end{array} \tag{10}$$

But the market economy approach imposes an additional restriction on the income allocation, specifically, that is takes one of the following two forms: (a) constant nominal income, $y^h(p) = y^h \in \mathbb{R}$ for all p, h, or (b) linear income, $y^h(p) = p^T \cdot \omega^h$, where $\omega^h \in \mathbb{R}^N$.

These conditions may be satisfied for special functional forms for individual utilities and the index. For instance, one can readily check that, if utilities are Cobb-Douglas and W is linear,

$$W\left(U^1, \ldots, U^H\right) = \sum_h \lambda_h U^h,$$

then the optimal consumption plan can be decentralized by allocating to agent h a fixed nominal income equal to $y_h = \lambda_h y$. Most of the time, however, the conditions are not satisfied; the relationship that exists, for given individual utilities $(U^1, \ldots, U^H)$, between the index W and the allocation $y(p) = [y^1(p), \ldots, y^H(p)]$ is in fact quite complex and in general is highly nonlinear.

3.2.2. The transferable utility case. An alternative situation in which the group's behavior boils down to a single utility maximization is when individual utilities exhibit a TU property (see, for instance, Browning et al. 2011). This happens when one can find, for

[5]The precise, formal argument is as follows. Consider an economy with H customers $U^1, \ldots, U^H$ and $(M+1)$ commodities, the M physical commodities plus money. There exists an initial (total) endowment of the $(M+1)$-th commodity (money) equal to y; regarding the other commodities, the initial endowment is nil, but they can be produced from the $(M+1)$-th commodity according to the linear production technology $y = \sum_k p_k x^k$. In this economy, by the second welfare theorem, any Pareto-efficient allocation can be decentralized as an equilibrium. Given the linear technology, equilibrium prices must be proportional to $(p_1, \ldots, p_M)$, and we can always normalize them to be equal to that vector. An equilibrium is uniquely characterized by the allocation of initial endowments $(y_1, \ldots, y_H)$, which we interpret as an income distribution within the group.

each agent h, a particular cardinalization such that, for all values of prices and income, the Pareto frontier is a hyperplane of the equation

$$\sum_h U^h = K$$

for some K that depends on prices and income. In other words, it must be the case that for some well-chosen cardinalization of individual preferences, agents are able to transfer utility between them at a constant exchange rate (which can be normalized to one).

When all goods are private, the TU obtains only for quasi-linear utilities:

$$U^h(x_h) = x_h^1 + u^h\left(x_h^2, \ldots, x_h^M\right).$$

Here the marginal utility of an additional dollar spent on private consumption of commodity 1 is always constant (and can be normalized to one). This form has strong (and unrealistic) implications; for instance, individual demands for all commodities but the first have a zero-income elasticity. Things become much more interesting when public goods are considered. Bergstrom & Cornes (1983) prove that the TU property obtains if and only if individual utilities can be put into a generalized quasi-linear form (possibly after an increasing transform and a renaming of the private goods):

$$U^h(X, x_h) = u^h\left(x_h^2, \ldots, x_h^M, X\right) + G(X)x_h^1, \tag{11}$$

where $G(X) > 0$ for all X. Note that the G function must be identical for all members, whereas the u functions can be specific to individuals. In words, the TU assumption implies that, for some well-chosen cardinalization of individual preferences, the marginal utility of an additional dollar spent on private consumption of commodity 1 is always the same for all members (although it need not be constant—it may vary with the vector of public goods).

Although this form remains constrained, the restrictions are much less stringent than the quasi-linear case (see Chiappori 2010 for a precise characterization of these restrictions). Interestingly, and similar to the market economy case, the main restriction affects the level of heterogeneity that is allowed between individual preferences: The function G, which determines the marginal utility of private commodity 1, must be the same for all agents in the group.

Under TU, the sole assumption of Pareto efficiency is sufficient to generate a representative consumer, at least when all agents consume a positive quantity of the first private commodity. Indeed, one can readily show that efficiency then requires that the group maximizes the sum of individual utilities. In particular, the level of all public and private consumption (other than of the first private good) is the same for all efficient outcomes. Thus, under TU and assuming efficiency, group members will agree on almost all consumption choices; the only conflict will be in how to divide the private good x^1, which is often referred to as money but may be interpreted more broadly as a medium of exchange. Lastly, if we define

$$U^G(X, x) = \max_{\sum_h x_h = x} \sum_{h=1}^{H} u^h\left(x_h^2, \ldots, x_h^M, X\right) + G(X)x^1,$$

then the group's aggregate demand (X, x) maximizes U^G under a budget constraint, and U^G is therefore the utility of the group's representative consumer.

The TU framework is extremely convenient for many economic problems and is therefore widely used.[6] Still, it comes at a cost. Because TU is compatible with the existence of a representative customer, the resulting behavior satisfies income pooling. As mentioned above, empirical evidence does not support this property. Moreover, the representation of group behavior it provides is highly peculiar: This is a world in which, under efficiency, group members do not disagree about anything except the allocation of one private good. Applied to household economics, this implies that parents must always agree on all public expenditures, from housing to health care and from the brand of a new car to the level of education to be provided to each child. Such a representation may sometimes be convenient; in many contexts, however, it conflicts with evidence and omits some of the most interesting issues of group behavior—namely how shifts in the members' respective powers affect the group's decisions and aggregate behavior. These are issues on which new approaches—and especially the collective model that we describe below—put a lot of emphasis, thus requiring a more general framework.

4. AGGREGATION IN MARKET ECONOMIES: NEW RESULTS, NEW PERSPECTIVES

The market economy approach was actively pursued in the 1970s and the 1980s. Since then, there have been new advances. First, the market aggregate demand problem, which had been open since Sonnenschein's (1973) formulation, was solved; several extensions, dealing primarily with the case of small groups, have subsequently been developed. Second, the standard interpretation of the DMS results—that general equilibrium theory has no empirical content—has been challenged, and a more subtle interpretation has emerged.

4.1. Aggregate Market Demand

Although Sonnenschein's first problem, the excess demand case, was solved within months, the second remained open for 25 years and was solved only in 1997.[7] The existence of a decomposition has so far only been proved locally (i.e., in some open neighborhood of a regular point), and only for analytic functions; moreover, the proof relies on one of the most impressive results of twentieth-century mathematics, the Cartan-Kähler theorem (see Kähler 1934, Cartan 1945, and Bryant et al. 1991 for a modern presentation).

We do not provide the entire proof here; the interested reader is referred to Chiappori & Ekeland (1997, 1999a,b, 2000, 2006, 2009a). Instead, we briefly present the mathematical nature of the problem and try to explain why the market demand problem turned out to be much more difficult than its apparently similar counterpart, the excess demand one.

We start by assuming that $y^b = 1$. This simplifies the notations without reducing the generality of the proof, which can readily be extended to any vector $y = (y^1, \ldots, y^H)$.

[6]According to Bergstrom (1989), it lies at the core of Becker's celebrated rotten kid theorem (see Browning et al. 2011 for a precise discussion).

[7]However, Andreu (1982) provides a solution for finite data sets, and Sonnenschein (1973) includes perceptive intuitions.

Also we assume that $H \geq N$. We consider some mapping $x(p)$ from $\mathbb{R}^N$ to itself, which has to be decomposed into the sum of H individual market demand functions,

$$x(p) = x_1(p) + \ldots + x_H(p), \tag{12}$$

such that for all h, $x_h(p)$ solves

$$\begin{gathered} \max U^h(x) \text{ subject to} \\ p^T x = 1, \end{gathered} \tag{13}$$

where U^h is smooth (in a sense that is discussed below), strongly convex, and strictly increasing.

As usual, the indirect utility of agent h is defined as the value of the program in Equation 13. By the envelope theorem,

$$D_p V^h = -\lambda^h x_h(p), \tag{14}$$

where λ^h is the Lagrange multiplier of the budget constraint in Equation 13. The problem thus becomes the following: Finding H smooth, decreasing, quasi-convex functions $V^1, \ldots, V^H$ such that the function $x(p)$ can be written as (the negative of) a convex combination of the gradients of the V^h,

$$x(p) = -\sum_h \frac{1}{\lambda^h} D_p V^h, \tag{15}$$

where the V^h satisfy the additional restriction

$$p^T D_p V^h = -\lambda_h. \tag{16}$$

Note that the market demand problem is similar to the excess demand one except for one feature—namely, the individual budget constraint is $p^T x = 1$ instead of $p^T z = 0$, so the condition on indirect utilities is Equation 16 instead of $p^T D_p V^h = 0$. This apparently minor variation results in a considerably more difficult problem. As mentioned above, an obvious but crucial property of a constraint like $p^T z = 0$ is that if it is satisfied by some function z, then it is also satisfied by $k \cdot z$ for any scalar function k—and the proof of the result heavily exploits this fact. No such property exists in the market demand context.

Decomposing a given function into a linear or convex combination of gradients is a standard problem in mathematics (often referred to as the Darboux problem; see, for instance, Ekeland & Nirenberg 2002). Here, however, two additional complexities appear. One is that the V functions must be quasi-convex; the other is that they must satisfy Equation 16. Unlike the excess demand case, these complexities cannot be overcome by simple manipulations; they require the full strength of the Cartan-Kähler approach. The same tools can actually be applied to the case of small economies, in which necessary and sufficient conditions can be derived (see, for instance, Ekeland & Djitte 2006).

Finally, it should be stressed that the class of mathematical problems just described—decomposing a given function into a convex combination of gradients, possibly under additional constraints—lies at the core of most, if not all, modern aggregation theory. It appears not only in the market problem, but also in the much more general approach presented in the next section below.

4.2. Is General Equilibrium Theory Testable?

A widely accepted interpretation of the DMS results is that they shed light on a severe weakness of general equilibrium theory, namely its inability to generate empirically falsifiable predictions. A prominent illustration of this stand is provided, for instance, by Arrow (1991, p. 201), who listed among the main developments of utility theory the result that "in the aggregate, the hypothesis of rational behavior has in general no implications," concluding that "if agents are different in unspecifiable ways, then . . . very few, if any, inferences can be made."

This view, however, has been recently challenged as overly pessimistic. New results show that general equilibrium theory can actually generate strong testable predictions, even for large economies. The main idea, initially introduced by Brown & Matzkin (1996), Snyder (1999), Brown & Shannon (2000), and Kubler (2003) and reformulated from a differential perspective by Chiappori et al. (2002a, 2004), can be summarized as follows. The DMS approach concentrates on the properties of aggregate excess (or market) demand as a function of prices. However, this viewpoint is not the most adequate to assess the testability of general equilibrium theory. As far as testable predictions are concerned, the structure of aggregate excess demand is not the relevant issue, if only because excess demand is, in principle, not observable, except at equilibrium prices, where, by definition, it vanishes. However, prices are not the only variables that can be observed to vary. Price movements reflect fluctuations of fundamentals, and the relationship between these fundamentals and the resulting equilibrium prices is the natural object for empirical observation. One of the goals of general equilibrium theory is to precisely characterize the properties of this relationship. As it turns out, this characterization generates strong testable restrictions.

To illustrate this view, Brown & Matzkin (1996) consider the simplest possible structure, namely an exchange economy. Here, for given preferences, the economy is fully described by the initial endowments, which are observable, in principle, and general equilibrium theory precisely describes the link between endowments and equilibrium prices by characterizing the structure of the equilibrium manifold. Brown & Matzkin derive a set of necessary and sufficient conditions in the form of linear equalities and inequalities that have to be satisfied by any finite data set consisting of endowments and equilibrium prices. They show that these relationships are indeed restrictive. Dealing with the same problem, Chiappori et al. (2002a, 2004) adopt a differentiable viewpoint; their necessary and sufficient conditions take the somewhat more familiar form of a system of partial differential equations, reminiscent of Slutsky conditions. In particular, these conditions can readily be imposed on a parametric estimation of the equilibrium manifold and therefore can be tested using standard econometric tools. They also show that these restrictions, if fulfilled, are sufficient to generically recover the underlying economy—including individual preferences. These results, however, require that individual endowments be observable; indeed, when only aggregate endowments are observable, a nontestability result can be proved.

The conclusion that emerges from this literature is that, in contrast to prior views, general equilibrium theory does generate strong, empirically testable predictions. The subtlety, however, is that tests can only be performed if data are available at the micro (here individual) level. One of the most interesting insights of new aggregation theory may be there—in the general sense that testability generally requires micro data and does not seem to survive (except maybe under stringent auxiliary assumptions) in a macro context, when only aggregates can be observed.

5. AGGREGATION IN THE SMALL: THE MICROECONOMICS OF EFFICIENT GROUP BEHAVIOR

A major development in aggregation theory has been the emergence of the so-called collective models of group behavior. Unlike the market economy literature developed in the 1970s and 1980s, these models mostly concentrate on small groups (formally defined as groups in which the number of agents is small relative to the number of commodities); therefore, some structure is preserved by aggregation. And unlike Samuelson's or Becker's approach, they do not try to force aggregate behavior into the unitary structure of consumer theory; on the contrary, they explicitly acknowledge that groups cannot be expected to behave as a single individual. The emphasis is actually put on what precisely distinguishes groups from individuals—that is, the existence of a (possibly complex) decision process, and more specifically the notion of power. Central to the collective approach is the view that power matters—that any variation in the allocation of power between members will systematically result in changes in the aggregate behavior of the group and that these changes constitute an extremely interesting object for economic analysis. In collective models, paying a benefit to the wife instead of the husband makes a difference, and this difference is a major topic of interest.

This perspective opens a host of new questions: How, and under which assumptions, should the decision process be modeled? How can we formally represent the abstract notion of power? Should the group remain a black box, or is there something one can say about its structure (utilities, decision process) from the sole observation of its aggregate behavior? Are empirical predictions possible, and of what kind? In what follows, we describe the answers provided by the main line of research in this direction. We first present the formal model. We then provide a full characterization of the aggregate demand functions stemming from this framework. Finally, we discuss issues related to identification; we show that, generically, a set of simple exclusion restrictions (one per group member) is sufficient to fully recover welfare allocation between members.

5.1. Efficiency and Power

The collective approach essentially relies on one basic assumption, namely efficiency. Whatever the decision process may be, it is assumed that it leads to efficient outcomes, in the usual (Pareto) sense that no alternative would have been preferred by all group members. Innocuous as it may seem, this assumption still excludes several existing models of group (often household) behavior based on noncooperative game theory, for instance. It also rules out asymmetric information or agency problems. As such, it is particularly relevant for modeling long-term interactions between members that know each other well (families being a typical example). More generally, it can be seen as a benchmark formulation that will be extended in the future. Also, it encompasses and generalizes both the market economy approach (as, in the latter setting, equilibria are Pareto efficient) and the unitary perspective à la Becker/Samuelson.

Formally, we thus assume the following.

> **Axiom 1 (efficiency):** The outcome of the group decision process is Pareto efficient; the consumption $(x_1, \ldots x_H, X)$ chosen by the group is such that no other vector $\left(\bar{x}_1, \ldots \bar{x}_H, \bar{X}\right)$ feasible at the same prices and incomes could make all members better off, one of them strictly so.

The set of Pareto-efficient allocations can be characterized in a number of equivalent ways. First, for any vector (π, y) of prices and income in $\mathbb{R}^{M+1}$, there must exist numbers $\bar{u}_2, \ldots, \bar{u}_H$ and vectors $X, x_1, \ldots, x_H$, which may depend on (π, y), such that $(X, x_1, \ldots, x_H)$ solves

$$\begin{gathered} \max_{X, x_1, \ldots, x_H} U^1(X, x_1, \ldots, x_H) \text{ subject to} \\ U^h(X, x_1, \ldots, x_H) \geq \bar{u}_h, \;\; h = 2, \ldots, H, \\ \pi^T \xi = y, \end{gathered} \tag{17}$$

where, again, $\pi = (P, p)$ and $\xi = \left(X, \sum_h x_h\right)$.

Second, if μ^h denotes the Lagrange multiplier of the h-th constraint, the axiom can be restated as follows: There exist $H - 1$ scalar functions $\mu^h(\pi, y) \geq 0$, $2 \leq h \leq H$, such that $(X, x_1, \ldots, x_H)$ solves

$$\begin{gathered} \max_{X, x_1, \ldots, x_H} \sum_h \mu^h U^h(X, x_1, \ldots, x_H) \quad \text{subject to} \\ \pi^T \xi = y, \end{gathered} \tag{18}$$

where $\mu^1 = 1$. The equivalence between efficiency and the maximization of a weighted sum of utilities is well known; the μ^h are the Pareto weights of the program. Clearly, Pareto weights are defined only up to some normalization. In the program given in Equation 18, the first weight is normalized to be one. Clearly, other normalizations are possible.

A more geometric interpretation is the following. For any given utility functions $U^1, \ldots, U^H$ and any price-income bundle, the budget constraint defines a Pareto set for the group (defined as the set of vectors $U^1, \ldots, U^H$ that are reachable); under the assumptions stated (concave utilities, convex production set), the Pareto set is moreover convex. From Axiom 1, the final outcome will be located on the frontier of the Pareto set. Under standard smoothness assumptions, this frontier is an $(H - 1)$-dimensional manifold, indexed by the vector $\mu = (1, \mu^2, \ldots, \mu^H)$.

An important remark is that the vector μ, normalized, for instance, by $\sum \mu_h = 1$, summarizes the decision process because it determines the final location of the demand vector on this frontier. In that sense, it describes the distribution of power within the group. If one of the weights, μ^h, is equal to one for every (π, y), then the group behaves as though h is the effective dictator. For intermediate values, the group behaves as though each person h has some decision power, and the person's weight μ^h can be seen as an indicator of this power. This power interpretation must be used with some care, as the Pareto coefficient μ^h depends on the particular cardinalization adopted for individual preferences; if U^h is replaced with $G(U^h)$ for some increasing mapping G, the set of Pareto-efficient allocations does not change, but the parameterization through the vector μ has to be modified accordingly. It follows that interpersonal comparisons of Pareto weights are meaningless; for instance, $\mu^h > \mu^r$ does not imply that h has more power than r. However, the variations of μ^h are significant, in the sense that for any fixed cardinalization, a policy change that increases μ^h while leaving μ^r constant unambiguously ameliorates the position of h relative to r.

If the μ^h are constant, then the program (P) boils down to the maximization of a unique utility under production and a budget constraint. We then get a variant of the Samuelson index model, and the group behaves as if it were a single decision maker. In general, however, the weights μ^h depend on prices and income because these variables in principle may influence the distribution of power within the group, hence the location of the final choice over the Pareto frontier. The maximand in P is therefore price dependent; the standard properties of unitary models do not apply in this context. However, the dependence on prices and income has a specific form, which is exploited in what follows.

Three additional remarks can be made. First, because we postulate throughout the absence of monetary illusion, the μ^h are taken to be zero homogeneous in (π, y). Second, following Browning & Chiappori (1998), we often add some structure by assuming that the μ^h are continuously differentiable. Third, if we assume that all commodities are privately consumed and there are no externalities, then by the second welfare theorem any Pareto-efficient allocation can be decentralized as an equilibrium—and we are back to the framework studied in Section 2. Indeed, the market economy approach is a special case of the collective model.[8]

5.2. Aggregate Demand of an Efficient Group: A Characterization

The characterization problem can be stated as follows. Take a group that satisfies the assumptions made above and that makes Pareto-efficient decisions under the constraints defined by its production technology and its budget. What restrictions (if any) on the aggregate demand function characterize the efficient behavior of the group, and how do these restrictions vary with the size of the group? In other words, is it possible to derive conditions that are sufficient for some demand function to stem from a Pareto-efficient decision process within a well-behaved group? Technically, consider a demand function $\xi = (X, x)$ of $(\pi, y) = (P, p, y)$ that satisfies two standard conditions, namely homogeneity and adding up (i.e., $\pi^T \xi = y$ for all π, y), and that is sufficiently smooth in a sense that is defined below. Are there necessary and sufficient conditions on ξ that stem from the theoretical structure under consideration, i.e., from the fact that it is the Pareto-efficient demand of an H-person group?

5.2.1. The SNR($H-1$) condition. We start with a set of necessary conditions that characterize group demand in the most general framework. In what follows, utilities are of the unrestricted form $U^h(X, x_1, \ldots, x_H)$—we simply assume that U^h is increasing and strongly concave; moreover, intragroup production could be introduced at no cost. We maintain the homogeneity assumption; therefore, we normalize y to be one. The budget constraint is

$$\pi^T \xi = 1,$$

and aggregate demand is now a function $\xi(\pi)$ of prices only.

[8]The collective approach also encompasses several models of household behavior that have been developed in the literature, including models based on cooperative bargaining (Manser & Brown 1980, McElroy & Horney 1981) or on equilibrium (Grossbard-Schechtman & Neuman 2003).

Household utility. As discussed above, Pareto efficiency requires that the group demand solves the program given in Equation 18 above. We define the function $\mathbb{U}^H$, from $\mathbb{R}^M \times \mathbb{S}$ to $\mathbb{R}$, where $\mathbb{S}$ denotes the H-dimensional simplex, by

$$\begin{gathered} \mathbb{U}^H(\xi,\mu) = \mathbb{U}^H\left(X, x, \mu^1, \ldots, \mu^H\right) = \max_{X,\, x1,\ldots,x_H} \textstyle\sum_h \mu^h U^h(X, x_1, \ldots, x_H) \\ \text{subject to } x = x_1 + \ldots + x_H. \end{gathered} \tag{19}$$

In words, $\mathbb{U}^H$ denotes the maximum value of the weighted sum $\sum_h \mu^h U^h$ when aggregate group demand is ξ. In that sense, $\mathbb{U}^H$ can be interpreted as the group's utility function, and Equation 18 is equivalent to maximizing U^H under the budget constraint

$$\begin{gathered} \max \mathbb{U}^H(\xi,\mu) \\ \text{subject to } \pi^T \xi = 1. \end{gathered} \tag{20}$$

In what follows, let $\tilde{\xi}(\pi,\mu)$ denote the solution to Equation 20.

It is crucial to remark that $\mathbb{U}^H$ also depends on the vector of Pareto weights $\mu = \left(\mu^1, \ldots, \mu^H\right) \in \mathbb{S}$. In particular, $\mathbb{U}^H$ is not a standard utility function: Because the μ^h are generally price and income dependent, so is $\mathbb{U}^H$. In practice, $\tilde{\xi}$, considered as a function of π only (for some fixed μ), is a standard demand function; as such, it satisfies Slutsky symmetry and negativeness. However, $\tilde{\xi}$ is not observable because one cannot vary π while keeping μ constant. What the econometrician observes (or may recover), i.e., the demand function ξ, is related to $\tilde{\xi}$ by

$$\xi(\pi) = \tilde{\xi}[\pi, \mu(\pi)]. \tag{21}$$

Slutsky matrix. We now define the Slutsky matrix associated with ξ by

$$S(\pi) = (D_\pi \xi)\left(I - \pi \xi^T\right).$$

This is the standard definition of a Slutsky matrix, adapted to take into account the normalization $y = 1$.[9] Note, incidentally, that $S(\pi)v = 0$ for all vectors $v \in \text{Span}\{\pi\}$. Indeed,

$$S(\pi)\pi = (D_\pi \xi)\left(\pi - \pi \xi^T \pi\right) = 0 \text{ because } \xi^T \pi = 1.$$

Now, from Equation 21, we see that

$$\begin{aligned} S(\pi) &= \left(D_\pi \tilde{\xi} + D_\mu \tilde{\xi} \cdot D_\pi \mu^T\right)\left(I - \pi \xi^T\right) \\ &= \left(D_\pi \tilde{\xi}\right)\left(I - \pi \xi^T\right) + D_\mu \tilde{\xi} \cdot D_\pi \mu^T \left(I - \pi \xi^T\right) \\ &= \Sigma(\pi) + R(\pi), \end{aligned}$$

where

$$\Sigma(\pi) = \left(D_\pi \tilde{\xi}\right)\left(I - \pi \xi^T\right) = \left(D_\pi \tilde{\xi}\right)\left(I - \pi \tilde{\xi}^T\right)$$

and

$$R(\pi) = D_\mu \tilde{\xi} \cdot D_\pi \mu^T \left(I - \pi \xi^T\right).$$

[9]Homogeneity implies by the Euler relation that $D_\pi \xi \cdot \pi + y D_y \xi = 0$ The Slustky matrix is defined as $S(\pi, y) = D_\pi \xi + D_y \xi \cdot \xi'$. Therefore, $S(\pi, y) = D_\pi \xi + \left(-\frac{1}{y} D_\pi \xi \cdot \pi\right) \cdot \xi'$, and for $y = 1$ the result obtains.

$\Sigma(\pi)$ is the Slustky matrix corresponding to the function $\tilde{\tilde{\xi}}(\cdot,\mu)$, as computed at $\mu(\pi)$. As such, it is symmetric and negative semidefinite and satisfies $v^T\Sigma(\pi)v = 0$ for all vectors $v \in \text{Span}\{\pi\}$. Moreover, the rank of $R(\pi)$ cannot exceed that of $(D_\pi\mu)$, which is at most $H-1$. We therefore can state the basic result from Browning & Chiappori (1998).

Proposition 1 [the SNR($H-1$) condition]: If the C^1 function $\xi(\pi)$ solves problem (P), then the Slutsky matrix $S(\pi) = (D_\pi\xi)\left(I - \pi\xi^T\right)$ can be decomposed as

$$S(\pi) = \Sigma(\pi) + R(\pi), \tag{22}$$

where (a) the matrix $\Sigma(\pi)$ is symmetric and satisfies $v^T\Sigma(\pi)v = 0$ for all vectors $v \in \text{Span}\{\pi\}$, $v^T\Sigma(\pi)v < 0$ for all vectors $v \notin \text{Span}\{\pi\}$, and (b) the matrix $R(\pi)$ is of rank at most $H-1$.

Equivalently, there exists a subspace $\mathcal{E}(\pi)$of dimension at least $M - H$ such that the restriction of $S(\pi)$ to $\mathcal{E}(\pi)$ is symmetric, negative definite, in the sense that $v^TS(\pi)w = w^TS(\pi)v$ and $v^TS(\pi)v < 0$ for all nonzero vectors $v, w \in \mathcal{E}(\pi)$.

Here, SNR $(H-1)$ stands for symmetric negative plus rank $(H-1)$. As discussed above, an appealing property of these conditions is that they stem from the most general version of the collective model; i.e., they do not require much beyond efficiency and differentiability. Also, the SNR$(H-1)$ property nicely generalizes the standard Slutsky symmetry of the unitary model. Indeed, when $H = 1$ (the unitary setting), then $R(\pi)$ is the null matrix, and $S(\pi) = \Sigma(\pi)$ is symmetric. In the general case in which $H \geq 1$, $S(\pi)$ needs not be symmetric, and $R(\pi)$ represents the deviation from symmetry. Then the rank of this deviation is at most the number of members minus one.

On a more technical side, the $(H \times M)$ matrix $D_\pi\mu^T\left(I - \pi\xi^T\right)$ can be written as

$$D_\pi\mu^T\cdot\left(I - \pi\xi^T\right) = \begin{pmatrix} v_1^T \\ v_2^T \\ \vdots \\ v_H^T \end{pmatrix},$$

where the vectors $v_1, \ldots, v_H \in \mathbb{R}^N$ are linearly dependent.[10] It follows that

$$\begin{aligned} R(\pi) &= D_\mu\tilde{\tilde{\xi}}\cdot\begin{pmatrix} v_1^T \\ v_2^T \\ \vdots \\ v_H^T \end{pmatrix} \\ &= \sum_h D_{\mu^h}\tilde{\tilde{\xi}}\cdot v_h^T = \sum_h u_h \cdot v_h^T, \end{aligned}$$

where

$$u_h = D_{\mu^h}\tilde{\tilde{\xi}}.$$

[10]Obviously, the vectors v_h vary with π and should be written $v_h(\pi)$. To simplify notations, we omit the reference to π whenever it can be done without ambiguity.

Also,

$$R(\pi) \cdot \pi = D_\mu \tilde{\xi} \cdot D_\pi \mu^T \left(I - \pi \xi^T\right) \cdot \pi = 0$$

because $\left(I - \pi\xi^T\right) \cdot \pi = \pi - \pi\left(\xi^T \pi\right) = \pi - \pi = 0$. Therefore, $v_h^T \cdot \pi = 0$ for $h = 1, \ldots, H$. By the same token,

$$S(\pi) \cdot \pi = \Sigma(\pi) \cdot \pi = 0.$$

Let $\mathcal{E}(\pi)$ denote the subspace orthogonal to $\{\pi, v_1, \ldots v_H\}$; its dimension is at least $M - H$. Then the space $\mathbb{R}^M$ can be decomposed as

$$\mathbb{R}^M = \mathrm{Span}\{\pi\} \oplus \mathrm{Span}\{v_1, \ldots, v_H\} \oplus \mathcal{E}(\pi),$$

and we know that for any two vectors v and w in $\mathcal{E}(\pi)$,

$$v^T S(\pi) w = v^T \Sigma(\pi) w = w^T \Sigma(\pi) v = w^T S(\pi) v$$

and

$$v^T S(\pi) v = v^T \Sigma(\pi) v < 0,$$

which shows that the restriction of S to $\mathcal{E}(\pi)$ is symmetric, negative definite, as stated in the last part of Proposition 1.

Geometric interpretation. A geometric interpretation of SNR$(H-1)$ is the following. Remember, first, that for any given H-tuple of utilities, the budget constraint defines the Pareto frontier as a function of the price-income bundle; then μ determines the location of the final outcome on the frontier. Under smoothness assumptions, the Pareto frontier is actually a manifold of dimension $H-1$. Assume now that prices and income are changed. This has two consequences. For one thing, the Pareto frontier will move. Keeping μ constant, this would change demand in a way described by the Σ matrix. However, this change will not violate Slutsky symmetry; that is, it is not different from the traditional, unitary effect. The second effect is that μ will also change; this will introduce an additional move of demand along the (new) frontier. This change (as summarized by the R matrix) does violate Slutsky symmetry. But moves along an $(H-1)$-dimensional manifold are quite restricted. For instance, the set of price-income bundles that lead to the same μ is likely to be quite large in general; indeed, under our smoothness assumption, it is an $(M-H-1)$-dimensional manifold. Considering the linear tangent hyperspace, this means that there is a whole linear manifold of codimension $(H-1)$ such that, if the (infinitesimal) change in prices and income belongs to that hyperplane, no deviation from Slutsky symmetry can be observed. In other words, the SNR$(H-1)$ condition is a direct consequence of the fact that, in an H-person household, the Pareto frontier is of dimension $H-1$, whatever the number of commodities.

Testing for SNR(H − 1). How can a property like SNR$(H-1)$ be tested ? The basic idea is that a matrix S is SNR$(H-1)$ if and only if the antisymmetric matrix $\mathcal{M} = S - S^T$ is of rank at most $2(H-1)$ (remember that a matrix $\mathcal{M}$ is antisymmetric if $\mathcal{M}^T = -\mathcal{M}$). A more precise statement is the following.

Lemma 1: Let S be some $SNR(H-1)$ matrix,

$$S = \Sigma + \sum_{h=1}^{H-1} u_h \cdot v_h^T,$$

where the vectors $(u_1, \ldots, u_H)$ are linearly dependent and the vectors $(v_1, \ldots, v_H)$ are linearly dependent. Then the matrix $\mathcal{M} = S - S^T$ is of rank at most $2(H-1)$, and $\text{Im}(\mathcal{M})$ (the subspace spanned by the columns of $\mathcal{M}$) is spanned by the vectors $(u_1, \ldots, u_H, v_1, \ldots, v_H)$.

Therefore, testing for the collective model amounts to testing for the rank of matrix $\mathcal{M} = (S - S^T)$. The collective model predicts this rank should be at most $2(H-1)$, while it would be zero in the unitary case (note that antisymmetry implies that the rank of $\mathcal{M}$ must be an even integer).

The tests just described are derived under the crucial assumption of efficiency. Alternative approaches have been developed; the reader is referred to Browning et al. (2011) for a detailed presentation. Lechene & Preston (2011) analyze the demand function of a couple stemming from a noncooperative model (involving private provision of the public goods) similar to that discussed in Section 4. They show that, again, a decomposition of the type SNR1 holds. However, the rank conditions on the deviation matrix R are different. Specifically, Lechene & Preston show that the rank of R can take any value between one and the number of public goods in the model. Recently, d'Aspremont & Dos Santos Ferreira (2009) introduced a general framework that provides a continuous link between the cooperative and the noncooperative solutions. In their setting, couples are characterized by a pair of parameters that indicate how cooperatively each agent behaves. Again, they derive an SNR$(H-1)$ decomposition; however, the rank of matrix R can now take values between one and twice the number of public goods.

Several tests of SNR$(H-1)$ have been empirically performed (Browning & Chiappori 1998, Dauphin & Fortin 2001, Dauphin 2003, Dauphin et al. 2008, Kapan 2009). They conclude that standard symmetry of the Slutsky matrix is strongly rejected for multiperson families, although quite interestingly it fails to be rejected for singles; moreover, SNR$(H-1)$ is not rejected for couples. Finally, one can use these approaches to assess the number of actual decision makers in the family (see Dauphin et al. 2008, Kapan 2009).

5.2.2. Sufficiency of the SNR$(H-1)$ condition. The condition SNR$(H-1)$ has been known to be necessary for some time. A more difficult question is with regard to sufficiency. Take a smooth demand function $\xi(\pi)$ that satisfies homogeneity, adding up, and SNR$(H-1)$. Can it be constructed as the aggregate demand of a Pareto-efficient group? Formally, the sufficiency problem thus can be stated as follows: Is it possible to find (*a*) functions $[x_1(\pi), \ldots, x_H(\pi), X(\pi)]$, (*b*) increasing, concave utility functions $U^1(x_1, \ldots, x_H, X), \ldots, U^H(x_1, \ldots, x_H, X)$, and (*c*) a vector function $\mu(\pi)$ in the H-dimensional simplex, such that $[\xi(\pi), x_1(\pi), \ldots, x_H(\pi), X(\pi)]$ solves the program given in Equation 18?

In other words, we are looking for an equivalent, in the collective setting, to the integrability theorem in the unitary case, whereby Slutsky conditions (with homogeneity and adding up) are sufficient for the existence of a well-behaved utility function generating the demand function under consideration.

We start with a simple methodological point, namely that to prove sufficiency, one has to prove the existence of only one set of utility and production functions, however simple. In particular, it suffices to prove sufficiency for the set of egoistic preferences of the form $U^h(x_h, X)$.

As it turns out, any demand that is (locally) compatible with the collective approach is compatible with the collective approach with egoistic preferences. Two caveats must be made, however. First, the proof requires some degree of smoothness of the demand; in practice, we assume that the function ξ is continuously differentiable. Second, the construction of individual utilities and Pareto weights is only local; i.e., we prove sufficiency in an open neighborhood of any regular point (in a sense that is precisely defined below). The global construction is still an open problem.

Our first task is to describe the basic mathematical structure of the identification problem. We start with introductory examples that show how the structure obtains in two specific but intuitive cases—namely, commodities are either all public or all private. We then address the general setting.

Two introductory examples. We now describe the mathematical structure of the problem in more detail. Our main conclusion is that some known function (aggregate demand, aggregate inverse demand, or a function derived from these) must be written as a convex combination of gradients. In other words, the key structure is the same as for the aggregate excess or market demand of a market economy, as discussed in the previous sections, despite the fact that the model is much more general.

We start with two simple examples that illustrate the main result in an intuitive way. For expository convenience, we disregard distribution factors for the moment.

Public goods only. We first consider a version of the model in which all commodities are publicly consumed (therefore, $\xi = X$ and $\pi = P$). Keeping the normalization $y = 1$, the program given in Equation 18 above can be written as

$$\begin{cases} \max_X \sum_h \mu^h(P) U^h(X) \\ P^T X = 1 \end{cases}. \tag{23}$$

Let $X^*(P)$ denote its solution. Assuming an interior solution, first-order conditions give

$$\sum_h \mu^h(P) D_X U^h(X) = \lambda(P) \cdot P, \tag{24}$$

where λ denotes the Lagrange multiplier of the budget constraint, and λ is a scalar function of P.

Next we assume that the Jacobian matrix $D_P X$ is of full rank on some open set. It follows that the function $X(P)$ is invertible, and we can define the inverse demand function $P(X)$. Then Equation 24 becomes

$$\sum_h \frac{\mu^h[P(X)]}{\lambda[P(X)]} D_X U^h(X) = P(X).$$

In this equation, the right-hand side is the known (inverse) demand function, whereas all functions in the left-hand side are unknown, and we want to prove their existence. The specific structure here is that the inverse demand function must be a linear combination of gradients of increasing, concave functions; moreover, the coefficients of the combination must be nonnegative. Note that when $H = 1$, this equation boils down to a well-known result, namely that the inverse demand function stemming from the maximization of a unique utility under a budget constraint must be proportional to the gradient of the utility function.

Finally, assume, conversely, that some given C^1 demand $X(P)$, satisfying $P^T \cdot X(P) = 1$, is regular in the sense just defined in some neighborhood and such that the inverse demand $P(X)$ can be written as

$$P(X) = \sum_h \bar{\mu}^h(X) D_X U^h(X),$$

where the $\bar{\mu}^h$ are positive, and the U^h are increasing and strongly concave. Define $\tilde{\mu}^h(P) = \bar{\mu}^h[X(P)]$ for $h = 1, \ldots, H$, and consider the program

$$\begin{cases} \max_X \sum_h^h \tilde{\mu}(P) U^h(X) \\ P^T X = 1 \end{cases}. \tag{25}$$

Because the maximand is strongly concave, the first-order conditions are sufficient for a global optimum. Hence $X(P)$ is the aggregate demand of the group thus defined.

Private goods only. The previous argument may seem specific to the public-good structure in which it was constructed. As it turns out, the underlying intuition is more general. To see why, let us briefly discuss an alternative polar case in which all commodities are privately consumed and individual utilities belong to the egoistic family. This case has been repeatedly studied in the literature, starting with Chiappori (1988a,b, 1992). Now the demand function $\xi(\pi)$ is in fact $x(p)$; the program is therefore

$$\begin{array}{c} \max_{x_1, \ldots, x_H} \sum_h \mu^h U^h(x_h) \quad \text{subject to} \\ p^T \left(\sum_h x_h \right) = 1, \end{array} \tag{26}$$

where y has again been normalized to one. Let $\left(x_1^*, \ldots, x_H^*\right)$ denote the solution to this program.

The notion of a sharing rule provides an equivalent but often more tractable version of this program. It relies on the following result.

> **Proposition 2**: There exist H scalar functions $\rho^1, \ldots, \rho^H$ of p, with $\sum_h \rho^h(p) = 1$, such that for any $h = 1, \ldots, H$, x_h^* solves
>
> $$\begin{array}{c} \max_{x_h} U^h(x_h) \quad \text{subject to} \\ p^T x_h = \rho^h(p). \end{array} \tag{27}$$
>
> **Proof**: Define $\rho^h = p^T x_h^*$, and assume that x_h^* does not solve Equation 27. Then there exists some $\bar{x}_h$ such that $p^T \bar{x}_h = p^T x_h^*$ and $U^h(\bar{x}_h) > U^h\left(x_h^*\right)$. But then the allocation $\left(x_1^*, \ldots, \bar{x}_h, \ldots, x_H^*\right)$ is feasible and Pareto dominates $\left(x_1^*, \ldots, x_H^*\right)$, a contradiction.

This is just a particular application of the second welfare theorem. Consider the group as a small, convex economy, in which all commodities $1, \ldots, N$ can be produced from a single input, money, according to the linear production technology $p^T\left(\sum_h x_h\right) = 1$. Then any Pareto-efficient allocation can be decentralized as an equilibrium; moreover, the linear technology requires that the prices within the economy be proportional to market prices p, hence the result.

In other words, when commodities are all private, an efficient allocation can always be seen as stemming from a two-stage decision process.[11] In the first stage, members decide on the allocation of total income $y = 1$ between them; member h receives ρ^h. In the second stage, agents each chose their vector of private consumption subject to their own budget constraints.

The vector $(\rho^1, \ldots, \rho^H)$ is the group's sharing rule. In a private-good context, the intragroup decision process is fully summarized by the sharing rule; in particular, there is a one-to-one mapping between (normalized) Pareto weights and the sharing rule. Moreover, this mapping is monotonic in the following sense: If we increase the Pareto weight of member i while keeping the other weights constant (possibly before renormalization), then the new sharing rule allocates more income to i than the initial one. A nice property of the sharing rule is that it does not depend on the particular cardinalization of individual utilities (it is expressed in dollars). The price to pay for this superior tractability is that sharing rules are less general, being defined for private goods only—although we extend the concept to a more general setting below.

Let $W^h(p)$ denote the value of the program given in Equation 27: It is called the collective indirect utility. It is defined as the utility reached by agent h, taking into account the intragroup decision process. If V^h denotes the standard, individual indirect utility of member h, we have

$$W^h(p) = V^h\left[p, \rho^h(p)\right]. \tag{28}$$

By the envelope theorem applied to the program given in Equation 27,

$$D_p W^h = \lambda^h\left(x_h - D_p\rho^h\right),$$

where λ^h is the Lagrange multiplier of the budget constraint, i.e., the marginal utility of the money of h. Therefore,

$$\sum_h \frac{D_p W^h}{\lambda^h} = \sum_h \left(x_h - D_p\rho^h\right).$$

Hence

$$\sum_h \frac{D_p W^h}{\lambda^h} = x(p),$$

as $\sum_h \rho^h(p) = 1$ implies $\sum_h D_p\rho^h(p) = 0$.

We can rewrite this equation in a slightly different way. Define $\tilde{W}(p) = -W(p)$; if $\alpha^h = 1/\lambda^h$, we have

$$-x(p) = \sum_h \alpha^h D_p^h \tilde{W}^h. \tag{29}$$

[11]Needless to say, we are not assuming that the actual decision process occurs in two stages. The result simply states that any efficient group behaves as if it were following a process of this type.

This time, it is the direct group demand function that is equal to a linear combination of gradients. Note that when $H = 1$, this equation boils down to the well-known Roy's identity, which states that a demand function stemming from the maximization of a unique utility under a budget constraint must be proportional to the gradient of the indirect utility [indeed, when $H = 1$, then $\rho(p) = 1$ and W is the standard indirect utility].

Conversely, assume that some smooth function $x(p)$, satisfying the Walras law, also satisfies Equation 29 in the neighborhood of some $\bar{p}$ for some positive α^h and some strictly decreasing, strongly convex $\tilde{W}^h$ (so that the W^h are strictly increasing and strongly concave). We now show that x can be decomposed as the aggregate demand of a group in which all commodities are privately consumed.

For each h, define a function $\rho^h(p)$ by

$$\rho^h(p) = p^T \cdot \left[D_p \rho^h - \alpha^h(p) D_p \tilde{W}^h \right]. \tag{30}$$

This is a linear first-order partial differential equation for $\rho^h(p)$. Note that the sum $\rho(p) = \sum \rho^h(p)$ satisfies a similar equation:

$$\rho = p^T D_p \rho + p^T x = p^T D_p \rho + 1, \tag{31}$$

which has the obvious solution $\rho(p) = 1$.

Equation 30 can be solved by the method of characteristics.[12] It follows that $\rho^h(p)$ can be prescribed arbitrarily on the affine hyperplane H defined as the set of p where $\bar{p}^T(p - \bar{p}) = 0$ (technically speaking, this is a noncharacteristic hypersurface, at least in some neighborhood of $\bar{p}$). We choose $\rho^h(p) = 1/S$ on H. It follows that $\rho = \sum \rho^h = 1$ on H, and as ρ satisfies Equation 31, $\sum \rho^h(p) = 1$ everywhere. As a consequence, we have

$$\sum D_p \rho^h = 0.$$

Now define

$$x_h(p) = D_p \rho^h - \alpha^h(p) D_p \tilde{W}^h. \tag{32}$$

We have

$$p^T x_h(p) = \rho^h(p), \quad 1 \leq h \leq S,$$
$$\sum_h x_h(p) = x(p).$$

We now have to show that the $x_h(p)$ solve the consumer's problem. For each h, consider the function

$$U^h(x) = \min_p \left\{ \tilde{W}^h(p) \,|\, p^T x \leq \rho^h(p) \right\}. \tag{33}$$

Note that, by the envelope theorem, U^h is differentiable and strictly increasing, and $D_x U^h[x_h(p)]$ is proportional to p. But Equation 32 is the optimality condition for this problem. Because $\tilde{W}^h$ is strongly convex, this condition is sufficient, so that

[12]In the case at hand, the method of characteristics consists of considering the flow $\frac{dp}{dt} = p$ in R^N, the solutions of which are given by $p(t) = p(0)e^t$. We also note that the function $\bar{\rho}^h(t) := \rho^h[p(t)]$ solves the differential equation $\bar{\rho}^h(t) = \frac{d\bar{\rho}^h}{dt}(t) - \alpha_s[p(t)]\, p(t)^T \cdot D_p \tilde{W}^h[p(t)]$ on R. This determines the solution $\bar{\rho}^h(p)$ on each trajectory of the flow (see, for instance, Bryant et al. 1991 for details).

$$U^h[x_h(p)] = \tilde{W}^h(p). \tag{34}$$

Now set

$$\bar{W}^h(p) = \sup_x \left\{ U^h(x) \mid p^T x \leq \rho^h(p) \right\}. \tag{35}$$

We have $\bar{W}^h(p) \geq U^h[x_h(p)] = \tilde{W}^h(p)$. Alternatively, for every x such that $p^T x \leq \rho^h(p)$, we have $U^h(x) \leq \tilde{W}^h(p)$. Taking the supremum with respect to all such x, we get $\bar{W}^h(p) \leq \tilde{W}^h(p)$. Finally $\bar{W}^h = \tilde{W}^h$, and Equation 35 becomes

$$\tilde{W}^h(p) = \max_x \left\{ U^h(x) \,|\, p^T x \leq \rho^h(p) \right\} = U^h[x_h(p)],$$

which tells us that $x_h(p)$ solves the consumer's problem for the utilities $U^h(x)$ and the sharing rule $\rho^h(p)$.

It remains to show that the U^h are quasi-concave, at least in some neighborhood of $\bar{p}$. To do this, pick x_1 and x_2 and a number a such that $U^h(x_1) \geq a$ and $U^h(x_2) \geq a$. We have

$$U^h\left(\frac{x_1 + x_2}{2}\right) = \min_p \left\{ \tilde{W}^h(p) \mid p^T \left(\frac{x_1 + x_2}{2}\right) \leq \rho^h(p) \right\}.$$

Now, if $\frac{1}{2} p^T x_1 + \frac{1}{2} p^T x_2 \leq \rho^h(p)$, then we must have $p^T x_i \leq \rho^h(p)$ for $i = 1$ or $i = 2$. Hence

$$\left\{ p \,|\, p^T \left(\frac{x_1 + x_2}{2}\right) \leq \rho^h(p) \right\} \subset \left\{ p \,|\, p^T x_i \leq \rho^h(p) \right\} \cup \left\{ p \,|\, p^T x_2 \leq \rho^h(p) \right\},$$

$$U^h\left(\frac{x_1 + x_2}{2}\right) \geq \min_{i=1,2} \left\{ \tilde{W}^h(p) \,|\, p^T x_i \leq \rho^h(p) \right\} = \min_{i=1,2} U^h(x_i) = a.$$

So the U^h are differentiable and quasi-concave.

The general case. In the two polar examples just considered—all goods are privately consumed, and all goods are publicly consumed—the sufficiency problem can thus be reformulated as follows: When can a given map from $\mathbb{R}^N$ to $\mathbb{R}^N$ be written as a linear combination of H gradients of increasing, strongly concave functions from $\mathbb{R}^N$ to $\mathbb{R}$? Specifically, this condition is necessary in both cases; furthermore, the condition is also sufficient, in the sense that whenever it is fulfilled one could construct a group for which the function at stake is indeed the aggregate demand.

We now show that this gradient structure is in fact general and that it fully characterizes the collective conditions. As explained above, it is sufficient to consider egoistic preferences without intragroup production. Therefore, we study the program

$$\begin{cases} \max_{x_1, \ldots x_H, X} \sum \mu^h(p, P) U^h(x_h, X) \\ p^T(x_1 + \ldots + x_H) + P^T X = 1. \end{cases} \tag{36}$$

Let $x_1(p, P), \ldots, x_H(p, P), X(p, P)$ denote its solution. The household demand function is then $\xi(p, P) = [x(p, P), X(p, P)]$, where $x = \sum_h x_h$.

In what follows, we repeatedly use the duality between private and public consumption, a standard tool in public economics. Assuming that the Jacobian matrix D_pX is of full rank, we consider the following change in variables:

$$\begin{aligned} \psi : \mathbb{R}^N &\rightarrow \mathbb{R}^N, \\ (p, P) &\rightarrow (p, X). \end{aligned} \tag{37}$$

The economic motivation for such a change in variables is clear. A basic insight underlying the duality between private and public goods is that, broadly speaking, quantities play for public goods the role of prices for private goods and vice versa. Intuitively, in the case of private goods, all agents face the same price but consume different quantities, which add up to the group's demand. With public goods, agents consume the same quantity, but face different (Lindahl) prices, which add up to the market price if the allocation is efficient. This suggests that whenever the direct demand function $x(p)$ is a relevant concept for private consumption, then the inverse demand function $P(X)$ should be used for public goods. The change of variable ψ allows us to implement this intuition.

In particular, instead of considering the demand function (x, X) as a function of (p, P), we often consider (x, P) as a function of (p, X) (then the public prices P are implicitly determined by the condition that the demand for public goods equals X while private prices equal p). Although these two viewpoints are clearly equivalent (one can switch form the first to the second and back using the change ψ), the computations are much easier (and more natural) in the second setting.

Conditional sharing rule. It is convenient, at this point, to introduce the notion of a conditional sharing rule, which directly generalizes the sharing rule introduced above in the case of private goods. It stems from the following result.

Lemma 2: For any given (p, P), let $\left(\bar{x}_1, \ldots, \bar{x}_H, \bar{X}\right)$ denote a solution to Equation 36. Define $\rho^h = p^T\bar{x}_h$ for $h = 1, \ldots, H$. Then for $h = 1, \ldots, H$, $\bar{x}_h$ solves

$$\begin{gathered} \max_{x_h} U^h(x_h, \bar{X}) \text{ subject to} \\ p^T x_h \leq \rho^h. \end{gathered} \tag{38}$$

Proof: Assume then there does not exist some $\tilde{x}_h$ such that $p^T\tilde{x}_h \leq \rho^h$ and $U^h(\tilde{x}_h, \bar{X}) > U^h(\bar{x}_h, \bar{X})$. But then the allocation $\left(\bar{x}_1, \ldots, \tilde{x}_h, \ldots \bar{x}_H, \bar{X}\right)$ is feasible and Pareto dominates $\left(\bar{x}_1, \ldots, \bar{x}_H, \bar{X}\right)$, a contradiction.

In words, an efficient allocation can be seen as stemming from a two-stage decision process. In the first stage, members decide on the public purchases X and on the allocation of the remaining income $y - P^TX$ between the members; member h receives ρ^h. In the second stage, agents each chose their vector of private consumption, subject to their own budget constraint and taking the level of public consumption as given. The vector $\rho = (\rho_1, \ldots, \rho_H)$ is the conditional sharing rule; it generalizes the notion of a sharing rule developed in collective models with private goods only because it is defined conditionally on the level of public consumption previously chosen. Of course, if all commodities are private ($K = 0$), then the conditional sharing rule boils down to the previous notion. In all cases, the conditional sharing rules satisfy the budget constraint

$$\sum_h \rho^h = 1 - P^TX. \tag{39}$$

As above, the conditional sharing rule can be expressed either as a function of (p, P) or, using the change in variable ψ, as a function of (p, X). We define the conditional indirect utility of member h as the value of the program given in Equation 27; hence

$$V^h(p, X, \rho) = \max\{U^h(x_h, X)\} \\ \text{subject to } p^T x_h = \rho, \tag{40}$$

which can be interpreted as the utility reached by member h when consuming X and being allocated an amount ρ for her private expenditures. Obviously, V^h is zero homogeneous in (p, ρ).

Collective indirect utility. Following Chiappori (2005) and Blundell et al. (2005), we introduce the following key definition, which again generalizes that introduced in the private-good case.

Definition 1: The collective indirect utility of agent h is defined by

$$W^h(p, X) = V^h\left[p, X, \rho^h(p, X)\right].$$

In words, W^h denotes the utility level reached by agent h, at prices p and with total income y, in an efficient allocation such that the household demand for public goods is X, taking into account the conditional sharing rule at stake. Note that W^h depends not only on the preferences of agent h (through the conditional indirect utility V^h), but also on the decision process (through the conditional sharing rule ρ^h). Hence W^h summarizes the impact on h of the interactions taking place within the group. As such, it is the main concept required for welfare analysis: Knowing each W^h, one can assess the impact of any reform (i.e., any change in prices and incomes) on the welfare of each group member. Also, in the case of public consumption only, W^h is simply equal to the direct utility U^h. Finally, remember that we are using the normalization $y = 1$. Without it, W^h would be a function of (p, X, y).

One can then prove (Chiappori & Ekeland 2006, 2009a) the following result: There exist scalar functions $\left(\gamma^1, \ldots, \gamma^h\right)$ such that

$$\sum\nolimits_h \gamma^h D_p W^h = -x - D_p A, \\ \sum\nolimits_h \gamma^h D_X W^h = P - D_X A, \tag{41}$$

where $A(p, X) = P(p, X)^T \cdot X$ denote the group's total expenditures on public goods. We thus see that in the general case under consideration, the sufficiency problem can be expressed as follows: Find a family of differentiable functions $W^h(p, X)$ on $\mathbb{R}^N$, each defined up to some increasing transform, such that the vector $\binom{-x-D_pA}{P-D_XA}$ can be expressed as a linear combination of the gradients of W^h.

The main result. We can now state the main result.

Theorem 1: Suppose a positive C^1 function $\xi(\pi)$ satisfies the Walras law $\pi^T \xi(\pi) = 1$ and condition SNR$(H-1)$ in some neighborhood of $\bar{\pi}$:

$$S(\pi) = (D_{\boldsymbol{\pi}}\boldsymbol{\xi})\left(I - \pi\xi^T\right) = \Sigma(\pi) + \sum_{h=1}^{H-1} a_h(\pi) b_h^T(\pi), \tag{42}$$

where $\Sigma(\pi)$ is symmetric, negative semidefinite, and the vectors $\xi(\pi)$, $a_h(\pi)$, and $b_h(\pi)$ are linearly independent. Then there are positive functions $\lambda_h(\pi)$ and increasing, strongly concave functions $V^h(\pi)$, $1 \leq h \leq H$, both defined on some neighborhood $\mathcal{N}$ of $\bar{\pi}$, such that the decomposition

$$\xi(\pi) = \sum_{h=1}^{H} \lambda_h(\pi) D_\pi V^h(\pi) \tag{43}$$

holds true on $\mathcal{N}$.

Proof: For the proof, the reader is referred to Chiappori & Ekeland (2009b).

In words, SNR($H-1$) is a necessary and sufficient local characterization of the aggregate demand of an efficient group. Some remarks are in order on that point. First, SNR($H-1$) remains necessary and sufficient even when one assumes either that all goods are publicly consumed or that all goods are privately consumed. In other words, the private versus public nature of intragroup consumption is not testable without additional assumptions. Second, the SNR($H-1$) condition is restrictive if and only if the number of commodities is larger than the number of agents. Indeed, in the opposite case, one can always write the decomposition in Equation 22 with $\Sigma(\pi) = 0$ and $R(\pi) = S(\pi)$. Quite interestingly, we confirm in this general framework an intuition already generated in the very specific case of a market economy—namely, that the individualistic foundations of the model induce some structure on the group's aggregate demand if and only if the group is small enough (technically, if it has fewer agents than commodities). Finally, note, however, that the key ingredient for this testability is Pareto efficiency. In that sense, the exclusive emphasis in the DMS literature on competitive equilibria in a market economy seems misleading ex post. Equilibria are but a specific form of Pareto-efficient allocations in a specific context (characterized by egoistic preferences, the absence of public goods and external effects), and the market economy literature in addition imposes highly specific types of intragroup allocation of income. The results just described imply that, perhaps surprisingly, none of these restrictions makes any difference for the basic conclusion.

5.2.3. Revealed preferences. The conditions described above characterize smooth demand functions and test for the generalized Slutsky conditions for integrability. An alternative approach to empirical demand analysis that has gained ground in the past few years is the RP approach derived from Afriat (1967) and Varian (1982). This style of analysis explicitly recognizes that we only ever have a finite set of observations on prices and quantities, which cannot be used to directly construct smooth demand functions without auxiliary assumptions. The RP approach instead identifies linear inequality conditions on the finite data set that characterizes rational behavior. The most-attractive feature of the Afriat-Varian approach is that no functional form assumptions are imposed. Moreover, powerful numerical methods are available to implement the RP tests. The drawback of the RP approach is that even when the data satisfy the RP conditions, we can only set identify preferences (see Blundell et al. 2008).

Chiappori (1988b) first generalized the unitary model RP conditions to the collective setting for a specific version of the collective model. The conditions for the general model have been established in Cherchye et al. (2007, 2008, 2009); these papers provide a complete characterization of the collective model in an RP context. This requires several significant extensions to the RP approach for the unitary model. For example, these authors allow for nonconvex preferences and develop novel (integer programming) methods because the linear programming techniques that work for the unitary model are not applicable to the collective model. The tests for collective rationality require that one find individual utility levels, individual marginal utilities of money (implying Pareto weights), and individual assignments for private goods and Lindahl prices for public goods. As in the unitary model, these methods can set identify only the preferences of the household members and the Pareto weight. Cherchye et al. (2011) apply these methods to a Russian expenditure panel.

5.3. Aggregate Demand of an Efficient Group: Identification

Broadly speaking, the identification question can be stated as follows: When is it possible to recover the underlying structure—namely, individual preferences, the effective distribution of power, and the resulting intragroup transfers—from the sole observation of the group's aggregate behavior?

Recent results in the literature on household behavior suggest that, surprisingly enough, when the group is small, the structure can be recovered under reasonably mild assumptions. For instance, in the model of household labor supply proposed by Chiappori (1988b, 1992), two individuals privately consume leisure and some Hicksian composite good. The main conclusion is that the two individual preferences and the decision process can generically be recovered (up to an additive constant) from the two labor supply functions. This result has been empirically applied, for example, by Fortin & Lacroix (1997) and Chiappori et al. (2002b) and extended by Chiappori (1997) to household production and by Blundell et al. (2007) and Donni (2003) to discrete participation decisions. Fong & Zhang (2001) consider a more general model in which leisure can be consumed both privately and publicly. Although the two alternative uses are not independently observed, in general they can be identified under a separability restriction, provided that the consumption of another exclusive good (e.g., clothing) is observed.

Taken together, these results suggest that multiperson groups need not remain black boxes, for which the structure cannot be investigated without precise information on intragroup decision processes. On the contrary, the group's aggregate behavior, as summarized by its demand function, contains potentially rich information on its structure—i.e., individual preferences and the distribution of powers between its members. We now substantiate this claim.

Define a structure as a set of individual utilities and Pareto weights (normalized, for instance, by the condition that their sum is one). Moreover, two structures $(U^1, \ldots, U^H; \mu_1, \ldots, \mu_H)$ and $(\bar{U}^1, \ldots, \bar{U}^H; \bar{\mu}_1, \ldots, \bar{\mu}_H)$ are equivalent if (*a*) for each h, there exists some increasing mapping F^h such that $U^h = F^h(\bar{U}^h)$, and (*b*) for any (π, y), $(\mu_1, \ldots, \mu_H)$ and $(\bar{\mu}_1, \ldots, \bar{\mu}_H)$ correspond to parameterizations of the same Pareto-efficient allocation for the respective cardinalizations of individual preferences; two structures are different if they are not equivalent.

A first result is the following.

> **Proposition 3**: In the most general version of the model, there exists a continuum of different structures that generate the same aggregate demand function. Moreover, the result remains valid even when all commodities are privately consumed or all commodities are publicly consumed.
>
> **Proof**: For the proof, the reader is referred to Chiappori & Ekeland (2009a,b).

In the most general case, there thus exists a continuum of observationally equivalent models—i.e., a continuum of structurally different settings generating identical observable behavior. This negative result implies that additional assumptions are required.

As it turns out, such assumptions are surprisingly mild. Essentially, it is sufficient that each agent in the group be excluded from the consumption of (at least) one commodity. We start with the case in which all commodities are publicly consumed. Then the following result holds.

> **Proposition 4**: In the collective model with H agents and public consumption only, if member 1 does not consume at least one good, then generically the utility of member 1 is exactly (ordinally) identifiable from household demand. If each member is excluded from the consumption of at least one specific good, then generically individual preferences are exactly (ordinally) identifiable from household demand, and for any cardinalization of individual utilities, the Pareto weights are exactly identifiable.
>
> **Proof**: For the proof, the reader is referred to Chiappori & Ekeland (2009a,b).

This result, in particular, has been applied to the collective formulation of household behavior. A large literature has been devoted to the analysis of labor supply, following the initial contribution of Chiappori (1988b, 1992). The idea is to consider the household as a two-person group making Pareto-efficient decisions on consumption and labor supply. Let L^h denote the leisure of member h, and w_h the corresponding wage. Various versions of the model can be considered. In each, Proposition 4 applies, leading to the full identifiability of the model (see Chiappori & Ekeland 2009b).

The general case (in which some goods are consumed privately and some publicly) is slightly more complex.

> **Proposition 5**: In the general, collective model with two agents, if each member is excluded from the consumption of at least one specific good, then generically the indirect collective utility of each member is exactly (ordinally) identifiable from household demand. For any cardinalization of indirect collective utilities, the Pareto weights are exactly identifiable.
>
> **Proof**: For the proof, the reader is referred to Chiappori & Ekeland (2009a,b).

Here, what is identified is the structure that is relevant to formulate welfare judgments (namely, the indirect collective utility W^h of each agent h). Remember that W^h is not identical to the standard indirect utility function V^h. The difference indeed is that W^h captures both the preferences of agent h (through V^h) and the decision process (which governs the way private commodities are allocated). In particular, identifying W^h is not equivalent to identifying V^h (hence U^h). If, for instance, all commodities are private, we have

$$W^h(p) = V^h\left[p, \rho^h(p)\right] = V^h\left[\frac{p}{\rho^h(p)}, 1\right], \tag{44}$$

and it is easy to prove that knowledge of W^h is not sufficient to independently identify both ρ^h and V^h. For any W^h, there exists a continuum of pairs (ρ^h, V^h) such that Equation 44 is satisfied.[13] In contrast to the public-good case, knowledge of the collective indirect utilities is therefore not sufficient, in the presence of private consumption, to identify individual preferences and the decision process (as summarized by the sharing rule). However, the indeterminacy is welfare irrelevant: Any welfare conclusion reached using one particular solution would remain valid for all the others (this is exactly the scope of the indirect collective utilities).

Finally, the previous identification result is only generic. One can find cases in which it does not obtain, but these cases are not robust to small perturbations.[14] Among these pathological contexts is the Samuelson index case, in which the group behaves as a single consumer. Intuitively, the basic condition (that some function must be decomposed as a linear combination of gradients) is then degenerate: The function is in fact proportional to a single gradient, which can itself be decomposed into a continuum of different sums. In other words, when a group behaves as a single consumer, then individual preferences are not identifiable. Ironically, a large fraction of the literature devoted to household behavior tends to assume a unitary setting, in which the group is described as a unique decision maker. Our conclusions show that this approach, although analytically convenient, entails a huge cost, as it precludes the (nonparametric) identification of individual consumption and welfare. In a general sense, nonunitary models are indispensable to address issues related to intragroup allocation.

6. CONCLUSION

The old literature on aggregation concentrated mainly on two issues. One was related to the structure of the aggregate (market or excess) demand of a large market economy; the other dealt with the conditions under which a small group would behave as a single decision maker. The research programs represented by these issues have mostly been completed. The questions raised by Sonnenschein (1973) have been answered (some quite recently), and Hildenbrand's contributions have illuminated how the aggregation of sufficiently heterogeneous individual behaviors could in fact create structure. Alternatively, the unitary representation of small groups (mostly families) has been the basis of a considerable theoretical and empirical literature.

Modern approaches have recently triggered a deep reconsideration of these views. The claim that general equilibrium theory could not generate testable predictions has been challenged; the consensus is now that testable implications exist, but they typically require micro data. What is dubious is that testable restrictions could be generated only if aggregate data are available, at least without very strong (and microempirically unrealistic) restrictions.

[13]For instance, pick up some arbitrary $\phi(p)$ mapping $\mathbb{R}^N$ into $\mathbb{R}$ and define an alternative solution $\left(\bar{\rho}^s, \bar{V}^s\right)$ by $\bar{\rho}^s(p) = \phi(p)\rho^s(p)$ and $\bar{V}^s(p, 1) = V^s[\phi(p)p, 1]$. Then Equation 44 is satisfied for the alternative solution.

[14]Technically, demands for which identification does not obtain must satisfy a specific partial differential equation (see Chiappori & Ekeland 2009b).

More importantly, the emphasis has shifted from aggregation in the large to aggregation in the small. Recent approaches have taken seriously the idea that the aggregate behavior of a (small) group exhibits specific features, which cannot in general and should not in any case be reduced to an individual decision process. These features actually raise fascinating issues about power relationships within groups and their impact on aggregate behavior, and a set of new results suggests that much can be learned about the former from a careful investigation of the latter. From this perspective, the macro fiction of a representative consumer no longer seems too attractive.

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