

## Topics on

### *Functional Analysis and Partial Differential Equations*

Universidad de La Habana, May 5<sup>th</sup> 2025 — May 16<sup>th</sup> 2025

The course is intended to present the main tools in the study of Partial Differential Equations used in the area of Mathematical Biology. There will be 20 hours of lectures and 20 hours of training sessions during which the students will do application exercises in order to get familiar with the notions presented in the course.

- 1) Functional analysis: Banach spaces, weak topology
- 2) Distributions, measures; derivatives in the sense of distributions; fundamental examples of distributions
- 3) Fourier transform  $\mathcal{F}$  in the space of Schwartz functions  $\mathcal{S}(\mathbb{R}^N)$ ; the space  $\mathcal{S}'(\mathbb{R}^N)$
- 4) Sobolev spaces  $H^s(\Omega)$ : definition, basic properties; trace of functions on the boundary; J.L. Lions' lemma about the trace
- 5) Lax-Milgram theorem, variational formulations of boundary value problems, the notion of weak solution
- 6) Laplace equation: fundamental solution, Green function, Dirichlet and Neumann problems, the Dirichlet principle, the maximum principle
- 7) Second order elliptic equations: existence of weak solutions, minimization procedure, regularity results
- 8) Eigenfunctions and eigenvalues of some elliptic operators
- 9) Evolution equations, semigroups generated by an elliptic operator, in particular the heat equation: fundamental solution, Cauchy problem, the non homogeneous equation, convergence to equilibrium

#### References:

- H. Brezis: *Analyse fonctionnelle*, Masson, (1983)
- L.E. Evans: *Partial Differential Equations*, GTM in Mathematics, vol. 19, AMS, (1998)
- D. Gilbarg & N.S. Trudinger: *Elliptic partial differential equations of second order*, Springer, (1977)
- O. Kavian: *Introduction à la Théorie des Points Critiques & Applications*, (chapitre 1), Springer, (1993)
- E. Lieb & M. Loss: *Analysis*, GTM in Mathematics, vol. 14, AMS, (1997)
- K. Yosida : *Functional Analysis*, éditions Springer-Verlag, 1980