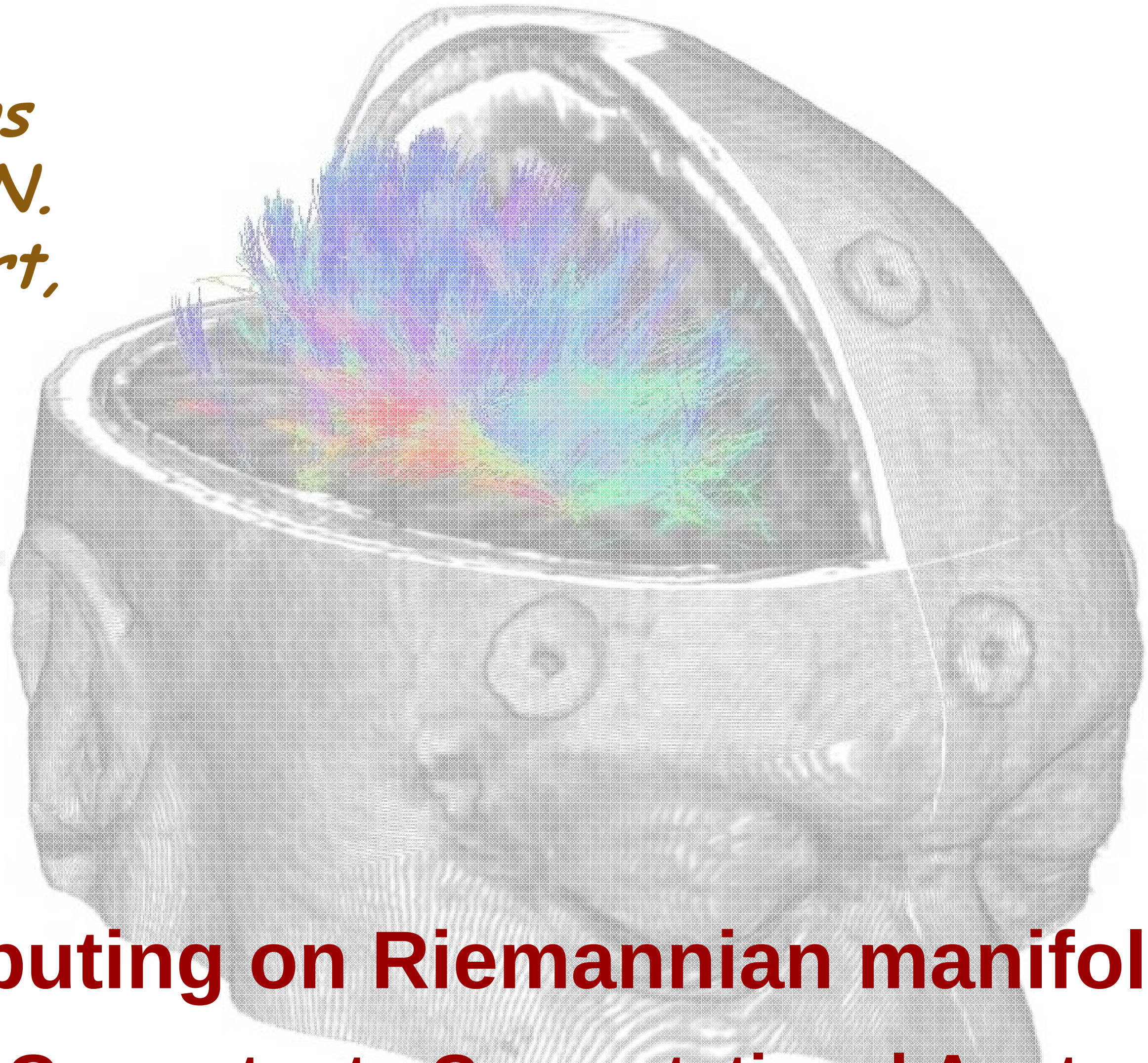
X. Pennec

With contributions from V. Arsigny, N. Ayache, J. Boisvert, P. Fillard, et al.



Statistical Computing on Riemannian manifolds

From Riemannian Geometry to Computational Anatomy

Mathematics and Image Analysis 2006

Standard Medical Image Analysis

Methodological / algorithmically axes

- Registration
- Segmentation
- Image Analysis/Quantification

Measures are geometric and noisy

- Feature extracted from images
- Registration = determine a transformations
- Diffusion tensor imaging

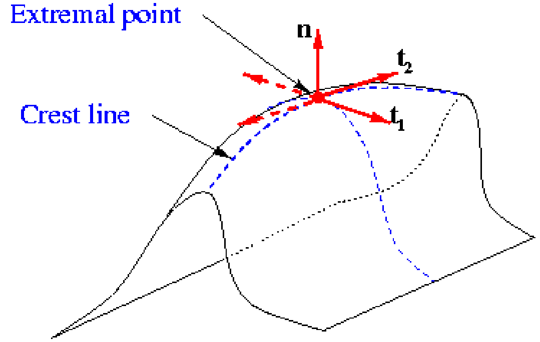
We need:

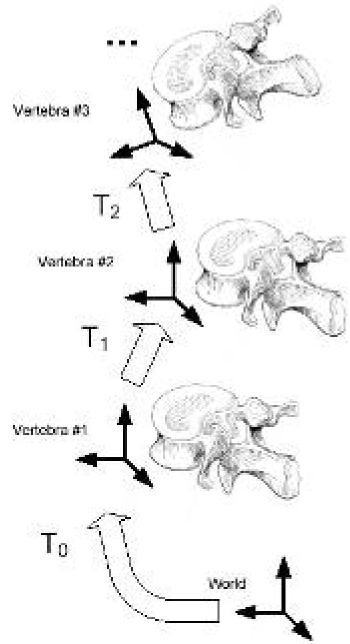
- Statistiques
- A stable computing framework

Historical examples of geometrical features

Geometric features

- Lines, oriented points…
- Extremal points: semi-oriented frames





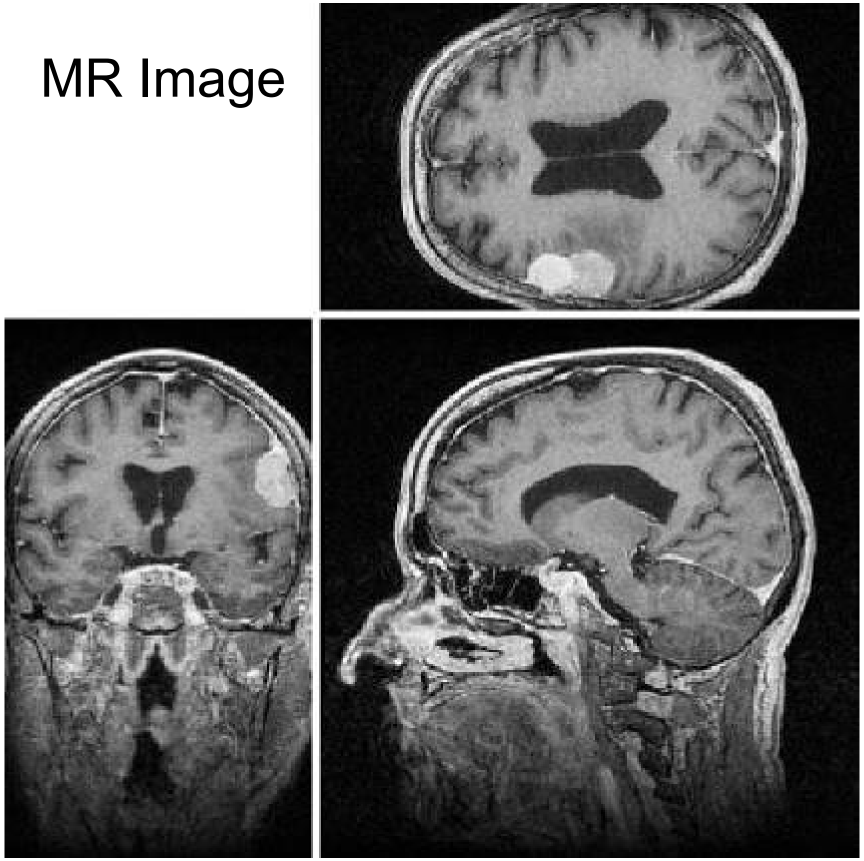
Transformations

- Rigid, Affine, locally affine, families of deformations

How to deal with noise consistently on these features?

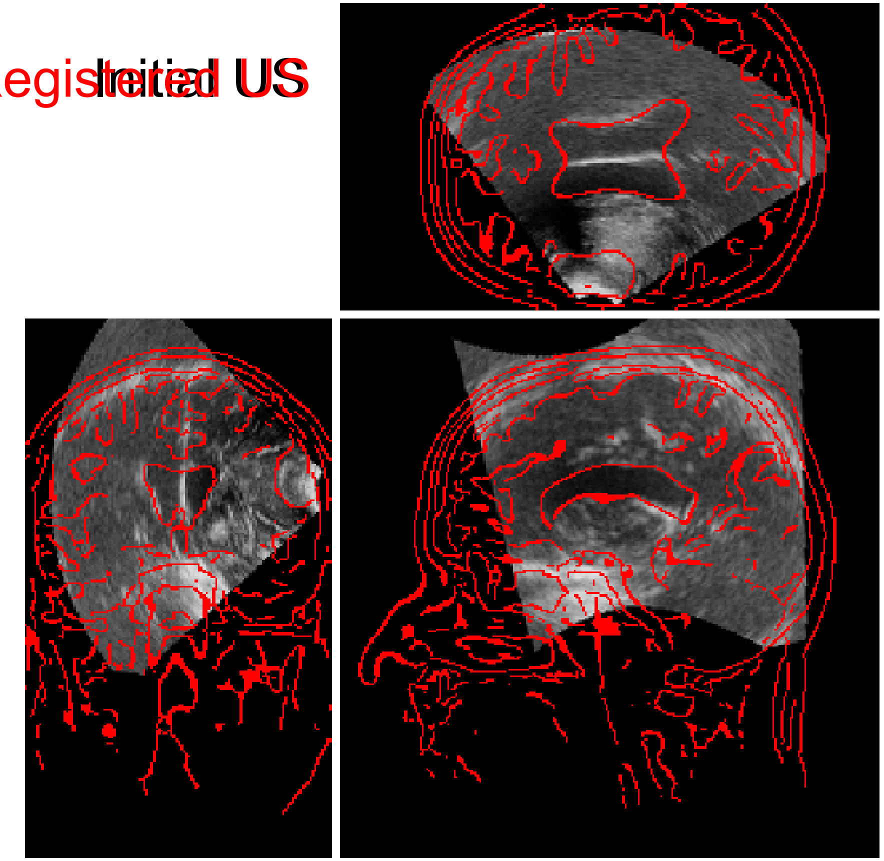
Per-operative registration of MR/US images

MR Image



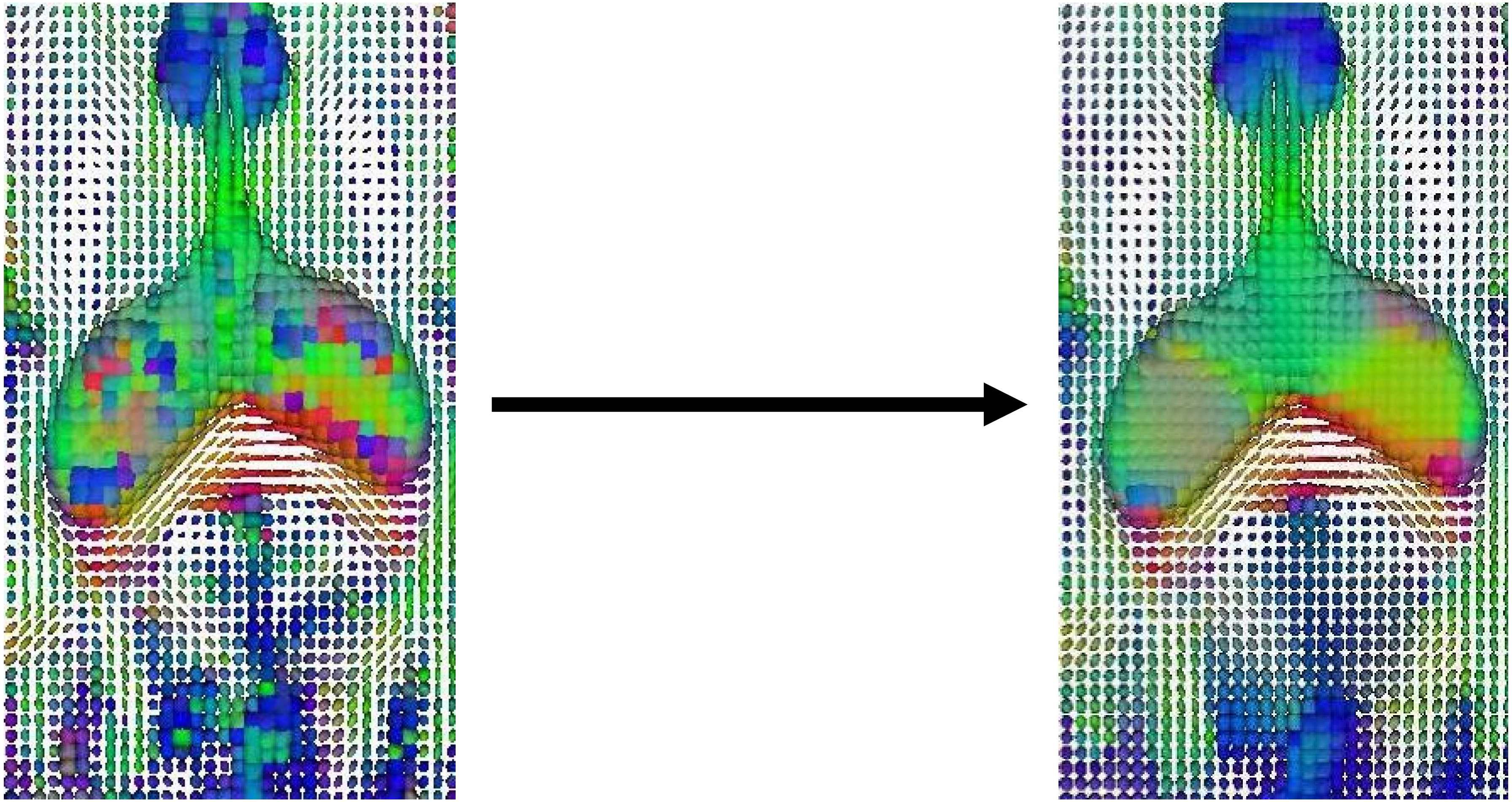
Initial US

Registered US



Performance Evaluation: statistics on transformations

Interpolation, filtering of tensor images

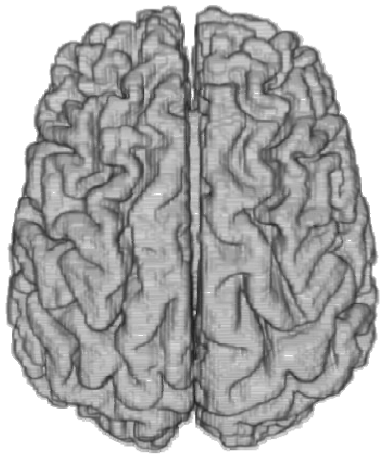
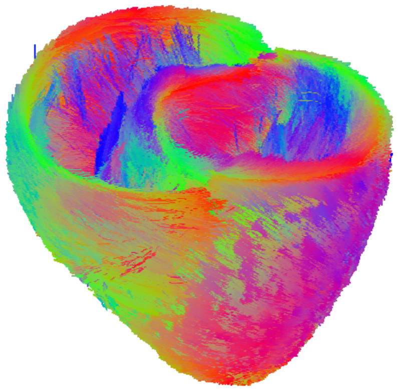
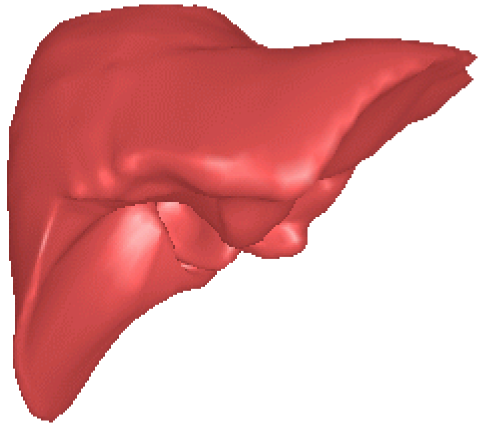


Raw

Anisotropic smoothing

Computing on Manifold-valued images

Computational Anatomy

***Computational Anatomy, an emerging discipline*, P. Thompson, M. Miller, NeuroImage special issue 2004**
***Mathematical Foundations of Computational Anatomy,* X. Pennec and S. Joshi, MICCAI workshop, 2006**

Modeling and Analysis of the Human Anatomy

- Estimate representative / average organ anatomies
- Model organ development across time
- Establish normal variability
- To detect and classify of pathologies from structural deviations
- To adapt generic (atlas-based) to patients-specific models

⇨Statistical analysis on (and of) manifolds

Overview

The geometric computational framework

⇨ (Geodesically complete) Riemannian manifolds

Statistical tools on pointwise features

- Mean, Covariance, Parametric distributions / tests
- Application examples on rigid body transformations

Manifold-valued images: Tensor Computing

- Interpolation, filtering, diffusion PDEs
- Diffusion tensor imaging

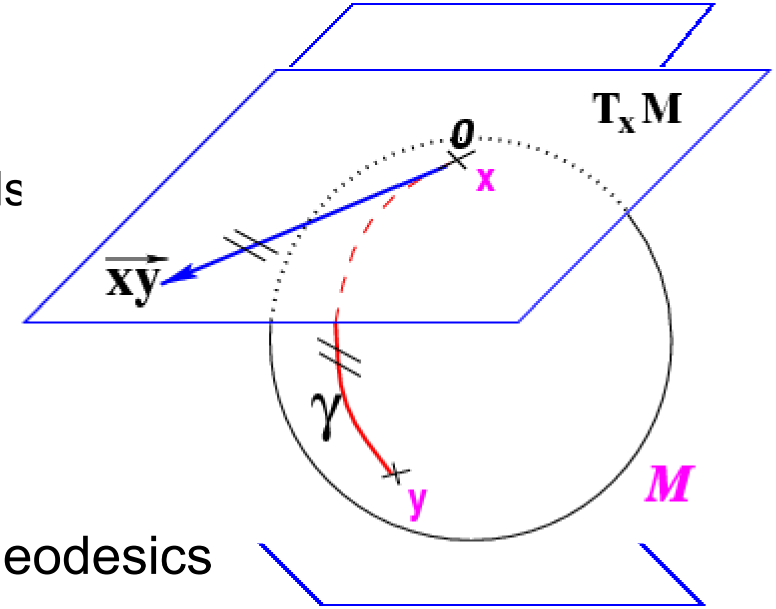
Metric choices for Computational Neuroanatomy

- Morphometry of sulcal lines on the brain
- Statistics of deformations for non-linear registration

Riemannian Manifolds: geometrical tools

Riemannian metric :

- Dot product on tangent space
- Speed, length of a curve
- Distance and geodesics
  - Closed form for simple metrics/manifolds
  - Optimization for more complex



Exponential chart (Normal coord. syst.) :

- Development in tangent space along geodesics
- Geodesics = straight lines
- Distance = Euclidean
- Star shape domain limited by the cut-locus
- Covers all the manifold if **geodesically complete**

Computing on Riemannian manifolds

Operation	Euclidean space	Riemannian manifold
Subtraction	$\overrightarrow{xy} = y - x$	$\overrightarrow{xy} = \log_x(y)$
Addition	$y = x + \overrightarrow{xy}$	$y = \exp_x(\overrightarrow{xy})$
Distance	$dist(x, y) = \lVert y - x \rVert$	$dist(x, y) = \lVert \overrightarrow{xy} \rVert_x$
Gradient descent	$\Sigma_{t+\varepsilon} = \Sigma_t - \varepsilon \nabla C(\Sigma_t)$	$\Sigma_{t+\varepsilon} = \exp_{\Sigma_t}(-\varepsilon \nabla C(\Sigma_t))$

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Statistical tools on Riemannian manifolds

Metric -> Volume form (measure) $d\mathcal{M}(\mathrm{x})$

Probability density functions $\forall X, P(x \in X) = \int_X p_{\mathbf{x}}(y).d\mathcal{M}(y)$

Expectation of a function ϕ from M into R :

- Definition : $\mathrm{E}\left[\phi(x)\right] = \int_{\mathcal{M}} \phi(y).p_{\mathbf{x}}(y).d\mathcal{M}(y)$
- Variance : $\sigma_{\mathbf{x}}^2(y) = \mathrm{E}\left[\mathrm{dist}(y, \underline{\mathrm{x}})^2\right] = \int_{\mathcal{M}} \mathrm{dist}(y,z)^2.p_{\mathbf{x}}(z).d\mathcal{M}(z)$
- Information (neg. entropy): $\mathrm{I}[\mathbf{x}] = \mathrm{E}\left[\log(p_{\mathbf{x}}(\mathbf{x}))\right]$

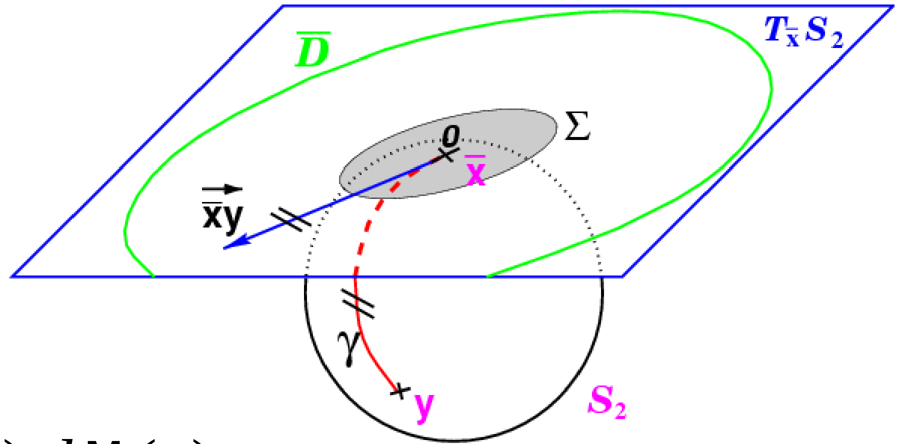
Statistical tools: Moments

Frechet / Karcher mean minimize the variance

$$\mathrm{E}[\mathbf{x}] = \underset{y \in \mathcal{M}}{\operatorname{argmin}} \left(\mathrm{E}\left[\operatorname{dist}(y, \mathbf{x})^2 \right] \right) \quad \Rightarrow \quad \mathrm{E}\left[\overrightarrow{\bar{x}\mathbf{x}} \right] = \int_{\mathcal{M}} \overrightarrow{\bar{x}\mathbf{x}} . p_{\mathbf{x}}(z) . d\mathcal{M}(z) = 0 \quad [P(C) = 0]$$

Geodesic marching

$$\bar{x}_{t+1} = \exp_{\bar{x}_t}(v) \quad \text{with} \quad v = \mathrm{E}\left[\overrightarrow{y\mathbf{x}} \right]$$



Covariance et higher moments

$$\Sigma_{\mathbf{xx}} = \mathrm{E}\left[\left(\overrightarrow{\bar{x}\mathbf{x}} \right) \left(\overrightarrow{\bar{x}\mathbf{x}} \right)^{\mathrm{T}} \right] = \int_{\mathcal{M}} \left(\overrightarrow{\bar{x}z} \right) \left(\overrightarrow{\bar{x}z} \right)^{\mathrm{T}} . p_{\mathbf{x}}(z) . d\mathcal{M}(z)$$

[Pennec, JMIV06, RR-5093, NSIP'99]

Distributions for parametric tests

Uniform density:

- maximal entropy knowing X $$p_{\mathbf{x}}(z) = \mathrm{Ind}_X(z) / \mathrm{Vol}(X)$$

Generalization of the Gaussian density:

- Stochastic heat kernel p(x,y,t) [complex time dependency]
- Wrapped Gaussian [Infinite series difficult to compute]
- Maximal entropy knowing the mean and the covariance

$$N(y) = k.\exp\left(\left(\overrightarrow{\bar{\mathbf{x}}\mathbf{x}}\right)^{\mathrm{T}} .\mathbf{\Gamma}.\left(\overrightarrow{\bar{\mathbf{x}}\mathbf{x}}\right) / 2 \right)$$

$$\mathbf{\Gamma} = \mathbf{\Sigma}^{(-1)} - \tfrac{1}{3}\mathrm{Ric} + O(\sigma) + \varepsilon(\sigma / r)$$

$$k = (2\pi)^{-n/2}.\det(\mathbf{\Sigma})^{-1/2}.\left(1 + O(\sigma^3) + \varepsilon(\sigma / r)\right)$$

Mahalanobis D2 distance / test: $$\mu_{\mathbf{x}}^2(\mathrm{y}) = \overrightarrow{\bar{\mathrm{x}}\mathrm{y}}^{t} .\Sigma_{\mathbf{xx}}^{(-1)} .\overrightarrow{\bar{\mathrm{x}}\mathrm{y}}$$

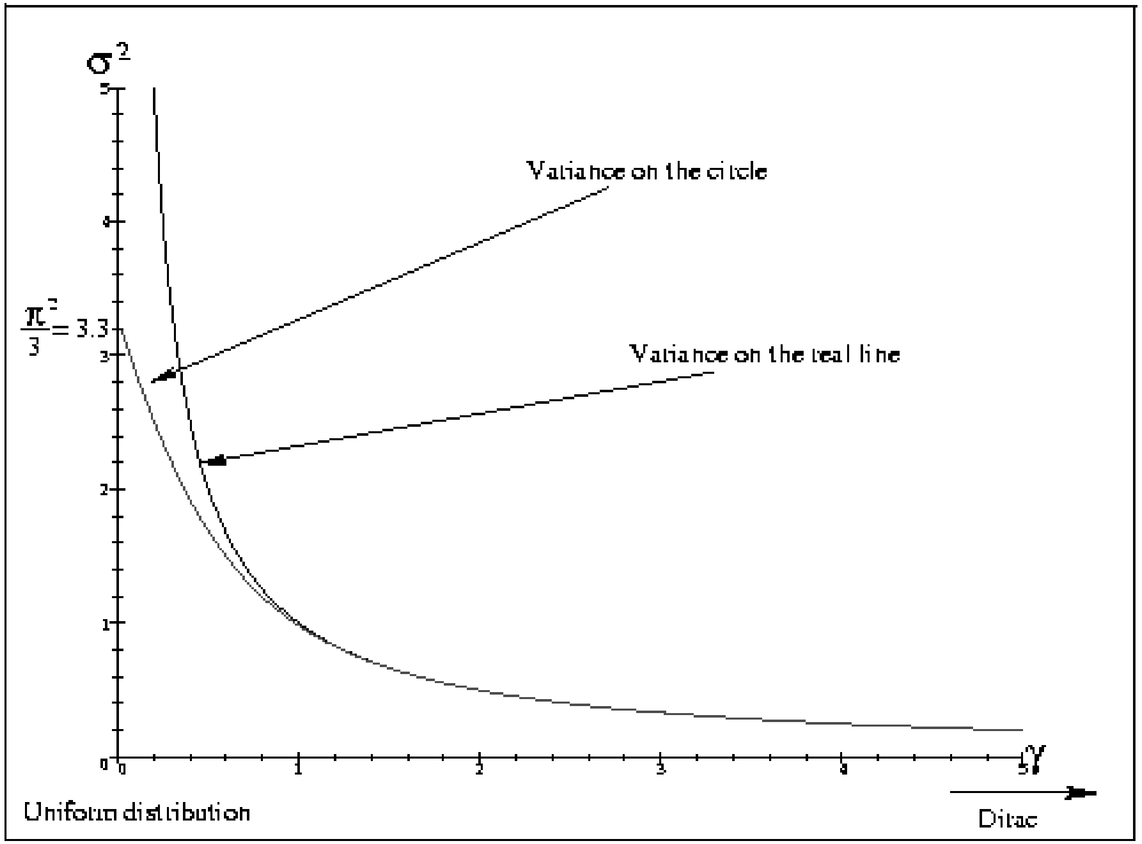
- Any distribution: $$\mathrm{E}\left[\mu_{\mathbf{x}}^2(\mathbf{x})\right] = n$$
- Gaussian: $$\mu_{\mathbf{x}}^2(\mathbf{x}) \propto \chi_n^2 + O(\sigma^3) + \varepsilon(\sigma / r)$$

[Pennec, JMIV06, NSIP'99]

Gaussian on the circle

Exponential chart: $x = r\theta \in \left]-\pi.r\,;\pi.r\right[$

Gaussian: truncated standard Gaussian



$r \to \infty$: standard Gaussian
(Ricci curvature → 0)

$\gamma \to 0$: uniform pdf with

$$\sigma^2 = (\pi.r)^2 / 3$$

(compact manifolds)

$\gamma \to \infty$: Dirac

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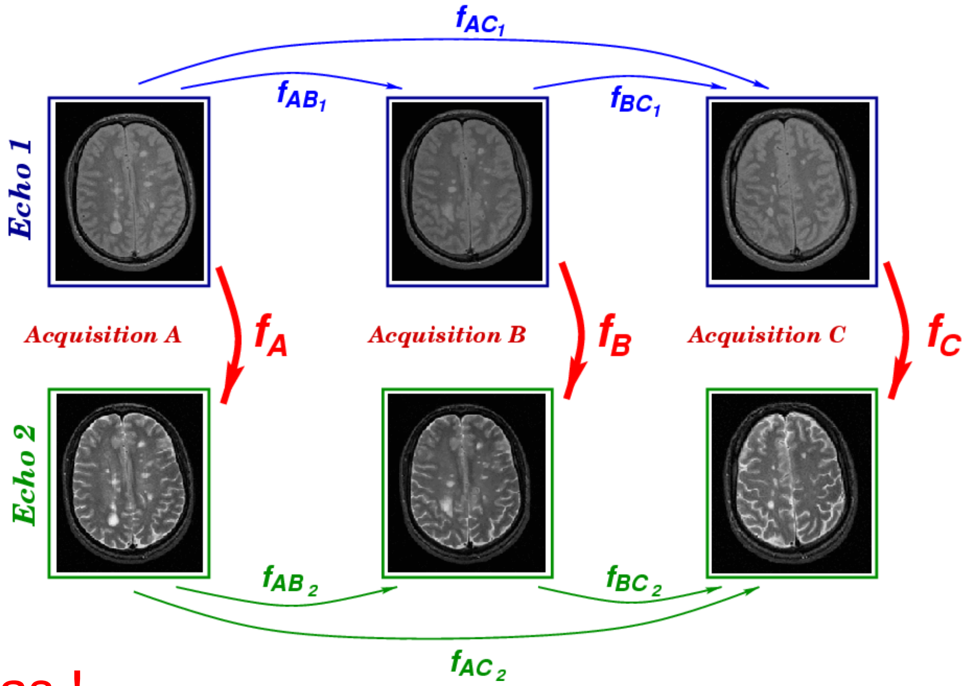
Validation of the error prediction

Comparing two transformations and their Covariance matrix :

$$\mu^2(T_1, T_2) \approx \chi_6^2$$

Mean: 6, Var: 12

KS test



Intra-echo: $\mu^2 \approx 6$, KS test OK

Inter-echo: $\mu^2 > 50$, KS test failed, Bias !

Bias estimation: (chemical shift, susceptibility effects)

- $\sigma_{rot} = 0.06$ deg (not significantly different from the identity)
- $\sigma_{trans} = 0.2$ mm (significantly different from the identity)

Inter-echo with bias corrected: $\mu^2 \approx 6$, KS test OK

[X. Pennec et al., Int. J. Comp. Vis. 25(3) 1997, MICCAI 1998]

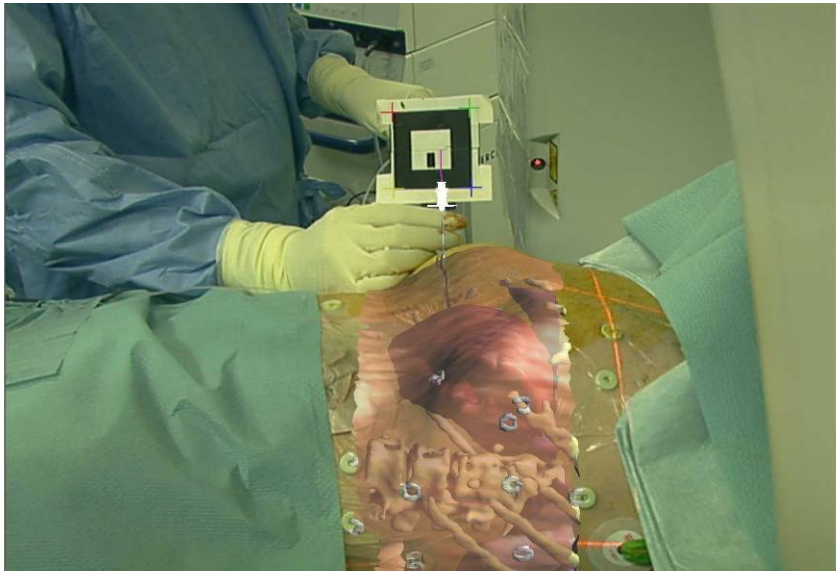
Liver puncture guidance using augmented reality

3D (CT) / 2D (Video) registration

- 2D-3D EM-ICP on fiducial markers
- Certified accuracy in real time

Validation

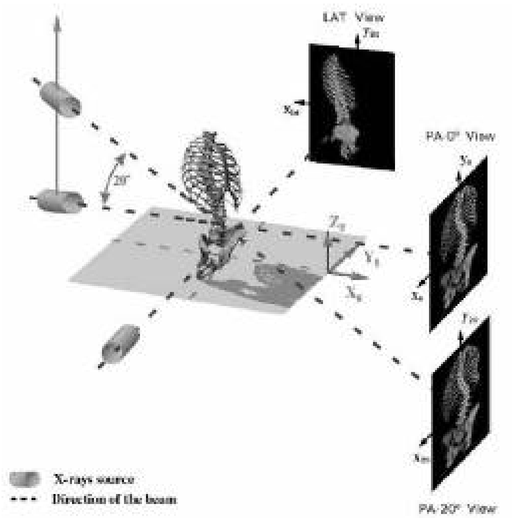
- Bronze standard (no gold-standard)
- Phantom in the operating room (2 mm)
- 10 Patient (passive mode): < 5mm (apnea)

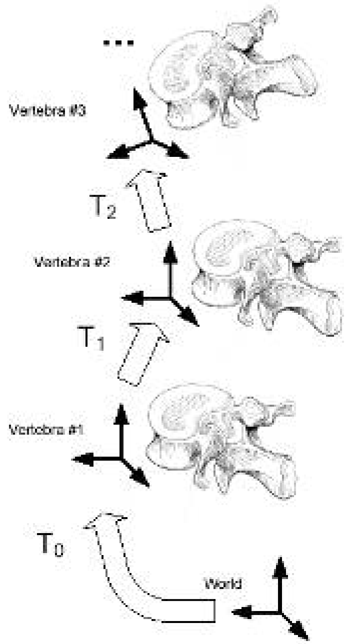


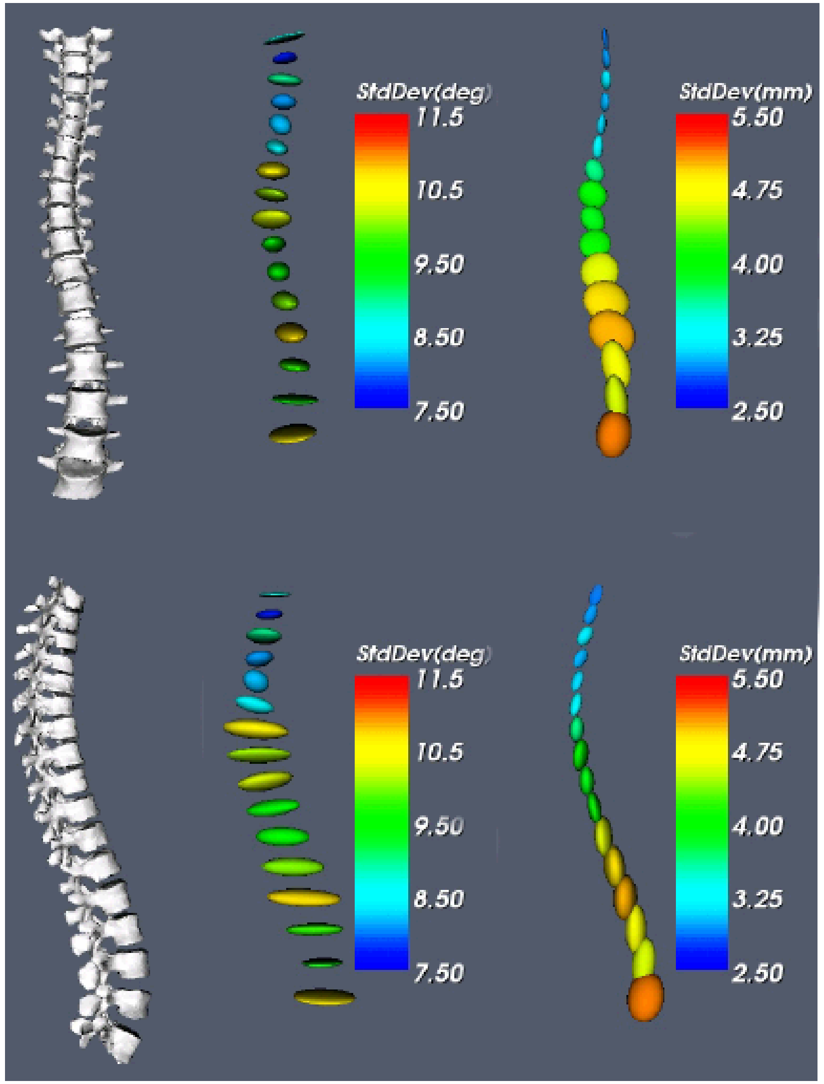
PhD S. Nicolau, MICCAI05, ECCV04, ISMAR04, IS4TM03, Comp. Anim. & Virtual World 2005, IEEE TMI (soumis)

Statistical Analysis of the Scoliotic Spine

[J. Boisvert, X. Pennec, N. Ayache, H. Labelle, F. Cheriet,, ISBI'06]







Database

- 307 Scoliotic patients from the Montreal's Sainte-Justine Hospital.
- 3D Geometry from multi-planar X-rays

Mean

- Main translation variability is axial (growth?)
- Main rotation var. around anterior-posterior axis

PCA of the Covariance

- 4 first variation modes have clinical meaning

Statistical Analysis of the Scoliotic Spine

- Mode 1: King's class I or III
- Mode 2: King's class I, II, III
- Mode 3: King's class IV + V
- Mode 4: King's class V (+II)

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Diffusion tensor imaging

Very noisy data

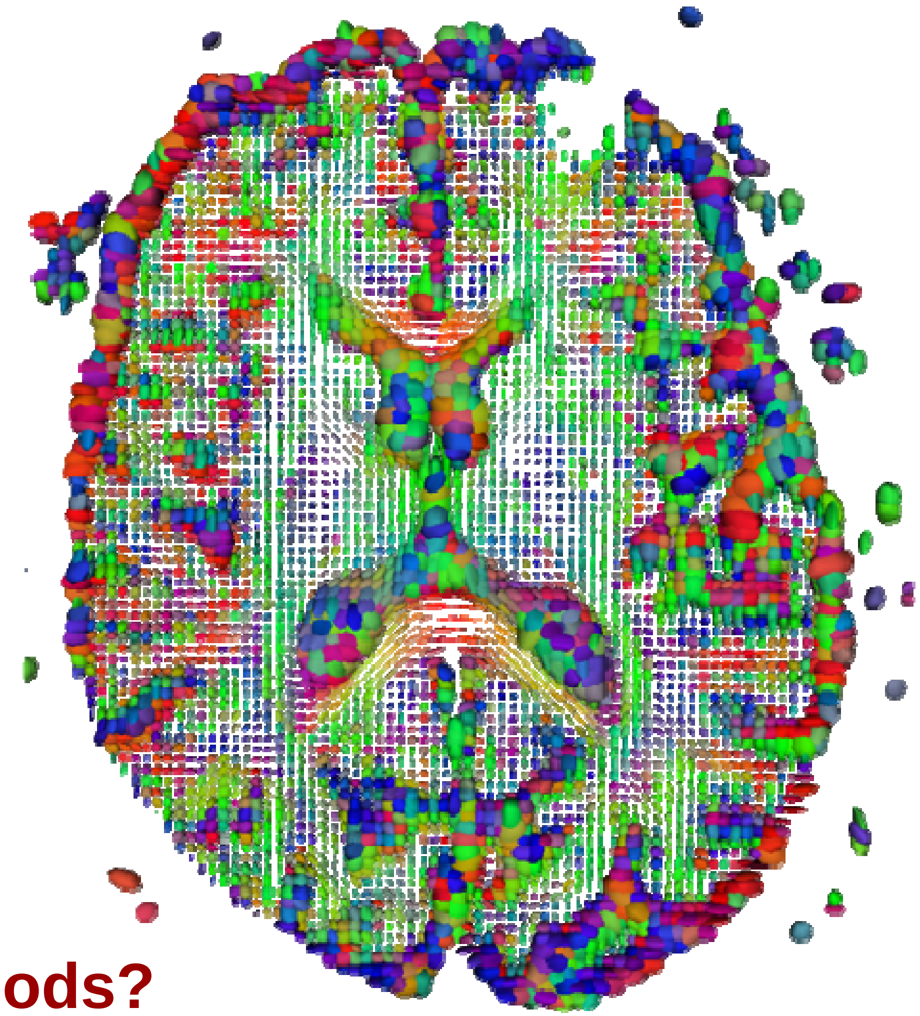
Preprocessing steps

- Filtering
- Regularization
- Robust estimation

Processing steps

- Interpolation / extrapolation
- Statistical comparisons

Can we generalize scalar methods?



DTI Tensor field (slice of a 3D volume)

Tensor computing

Tensors = space of positive definite matrices

- Linear convex combinations are stable (mean, interpolation)
- More complex methods are not (null or negative eigenvalues) (gradient descent, anisotropic filtering and diffusion)

Current methods for DTI regularization

- Principle direction + eigenvalues [Poupon MICCAI 98, Coulon Media 04]
- Iso-spectral + eigenvalues [Tschumperlé PhD 02, Chef d'Hotel JMIV04]
- Choleski decomposition [Wang&Vemuri IPMI03, TMI04]
- Still an active field…

Riemannian geometric approaches

- Statistics [Pennec PhD96, JMIV98, NSIP99, IJCV04, Fletcher CVMIA04]
- Space of Gaussian laws [Skovgaard84, Forstner99,Lenglet04]
- Geometric means [Moakher SIAM JMAP04, Batchelor MRM05]
- Several papers at ISBI'06

Affine Invariant Metric on Tensors

Action of the Linear Group GL_n

$$A * \Sigma = A.\Sigma.A^T$$

Invariant distance

$$dist(A*\Sigma_1, A*\Sigma_2) = dist(\Sigma_1, \Sigma_2)$$

Invariant metric

$$\langle W_1 | W_2 \rangle_\Sigma \overset{def}{=} \langle \Sigma^{-1/2} * W_1, \Sigma^{-1/2} * W_2 \rangle_{Id}$$

- Usual scalar product at identity $\langle W_1 | W_2 \rangle_{Id} \overset{def}{=} Tr\left(W_1^T W_2\right)$
- Geodesics $\exp_\Sigma(\overrightarrow{\Sigma\Psi}) = \Sigma^{1/2} \exp(\Sigma^{-1/2}.\overrightarrow{\Sigma\Psi}.\Sigma^{-1/2})\Sigma^{1/2}$
- Distance $dist(\Sigma, \Psi)^2 = \langle \overrightarrow{\Sigma\Psi} | \overrightarrow{\Sigma\Psi} \rangle_\Sigma = \left\| \log(\Sigma^{-1/2}.\Psi.\Sigma^{-1/2}) \right\|_{L_2}^2$

[X Pennec, P.Fillard, N.Ayache, IJCV 66(1), Jan. 2006 / RR-5255, INRIA, 2004]

Exponential and Logarithmic Maps

Geodesics $\Gamma_{Id,W}(t) = \exp(tW)$

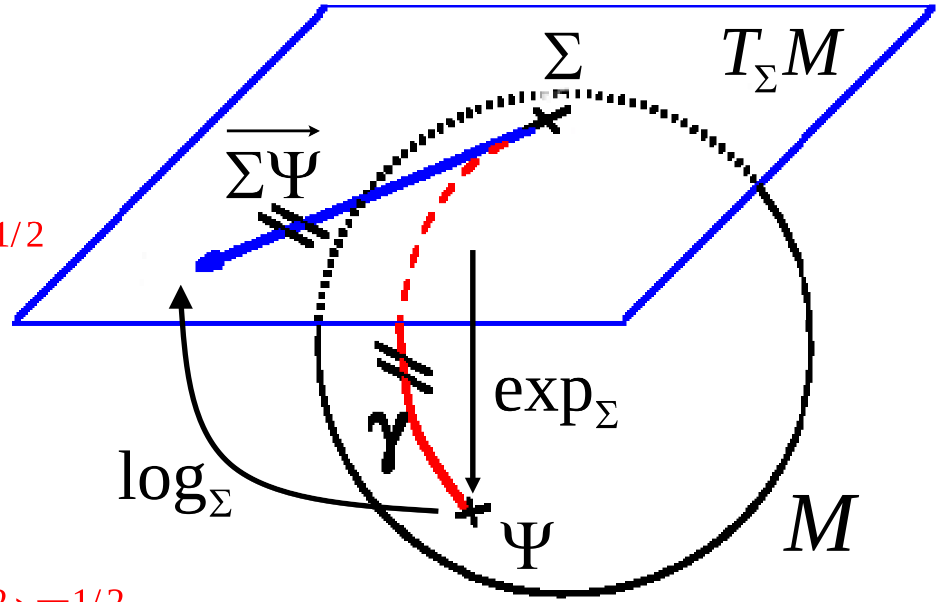
- Exponential Map :

$$\exp_\Sigma(\overrightarrow{\Sigma\Psi}) = \Sigma^{1/2} \exp(\Sigma^{-1/2}.\overrightarrow{\Sigma\Psi}.\Sigma^{-1/2})\Sigma^{1/2}$$

- Logarithmic Map :

$$\overrightarrow{\Sigma\Psi} = \log_\Sigma(\Psi) = \Sigma^{1/2} \log(\Sigma^{-1/2}.\Psi.\Sigma^{-1/2})\Sigma^{1/2}$$

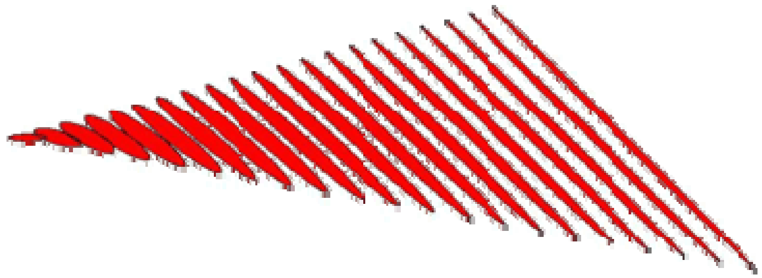
$$dist(\Sigma,\Psi)^2 = \left\langle \overrightarrow{\Sigma\Psi} \middle| \overrightarrow{\Sigma\Psi} \right\rangle_\Sigma = \left\| \log(\Sigma^{-1/2}.\Psi.\Sigma^{-1/2}) \right\|_{L_2}^2$$

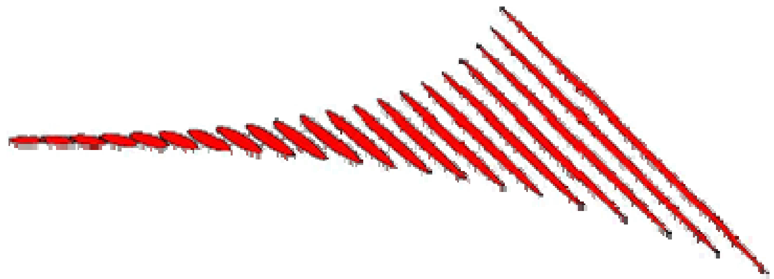


Tensor interpolation

Geodesic walking in 1D

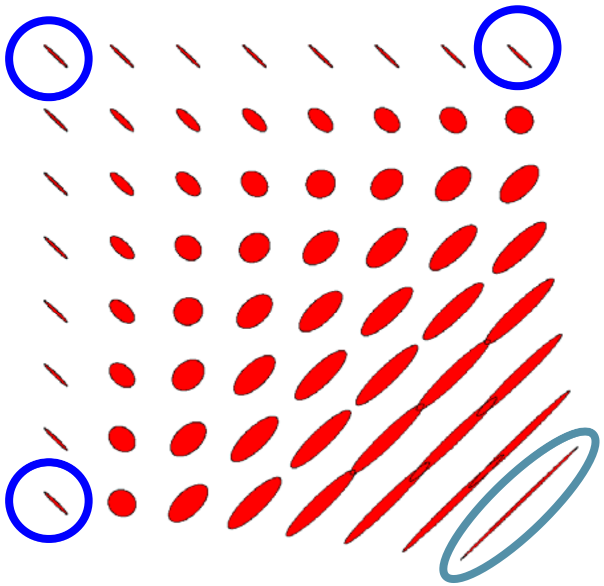
$$\Sigma(t) = \exp_{\Sigma_1}(t\overrightarrow{\Sigma_1\Sigma_2})$$

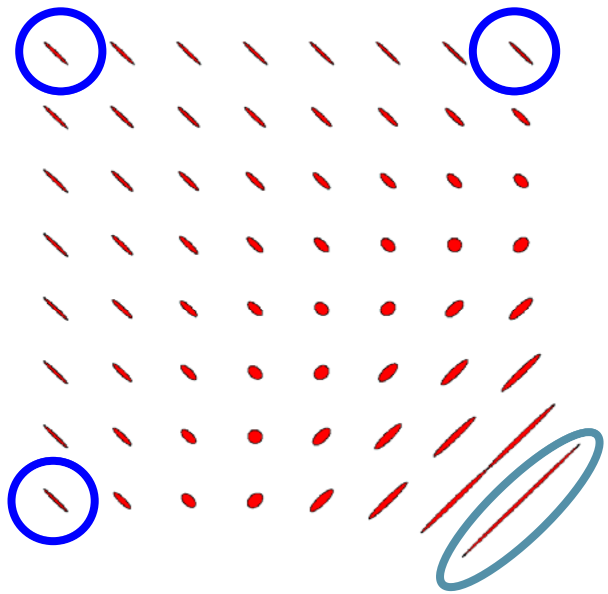




Weighted mean in general

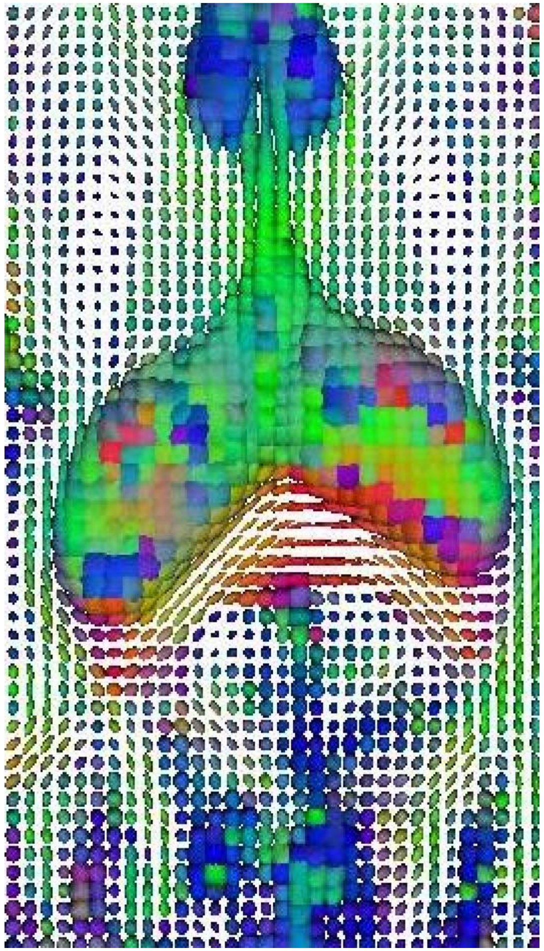
$$\Sigma(x) = \min_{\Sigma} \sum w_i(x) \;\; dist(\Sigma, \Sigma_i)^2$$





Gaussian filtering: Gaussian weighted mean

$$\Sigma(x) = \min \sum_{i=1}^{n} G_\sigma(x - x_i)\; dist\;(\Sigma, \Sigma_i)^2$$



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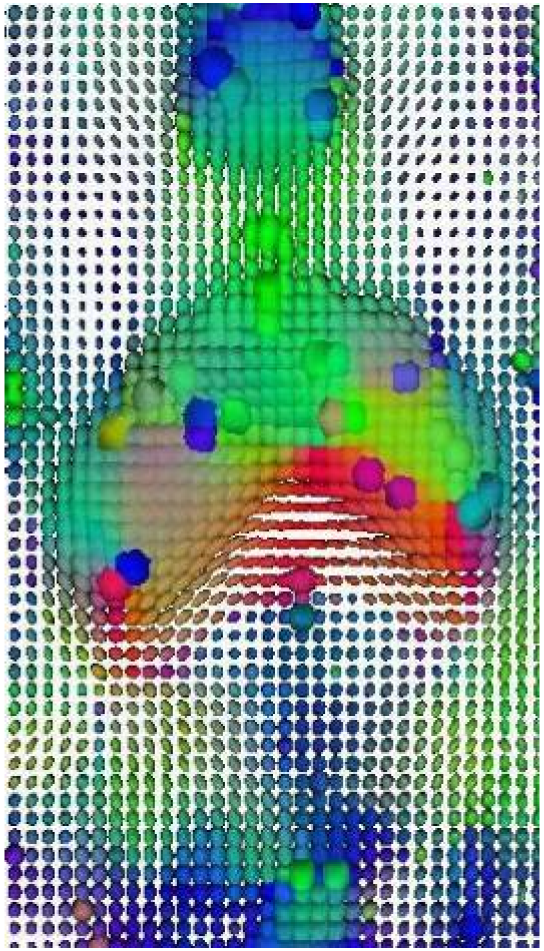
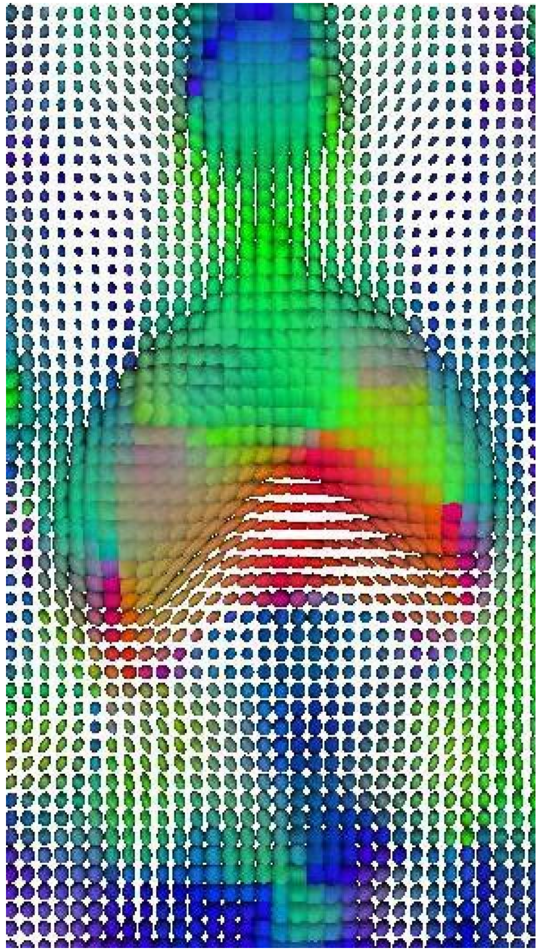


Coefficients σ=2



Riemann σ=2

PDE for filtering and diffusion

Harmonic regularization

$$C(\Sigma) = \int_{\Omega} \left\| \nabla \Sigma(x) \right\|_{\Sigma(x)}^2 dx$$

- Gradient = manifold Laplacian $\nabla C(x) = -2 \Delta \Sigma(x)$

$$\Delta \Sigma(x) = \sum_i \partial_i^2 \Sigma - \sum_i (\partial_i \Sigma) \Sigma^{(-1)} (\partial_i \Sigma) = \sum_u \frac{\overrightarrow{\Sigma(x)\Sigma(x+u)}}{\|u\|^2} + O\left(\|u\|^2\right)$$

- Integration scheme = geodesic marching

$$\Sigma_{t+1}(x) = \exp_{\Sigma_t(x)}\left(-\varepsilon \nabla C(\Sigma)(x)\right)$$

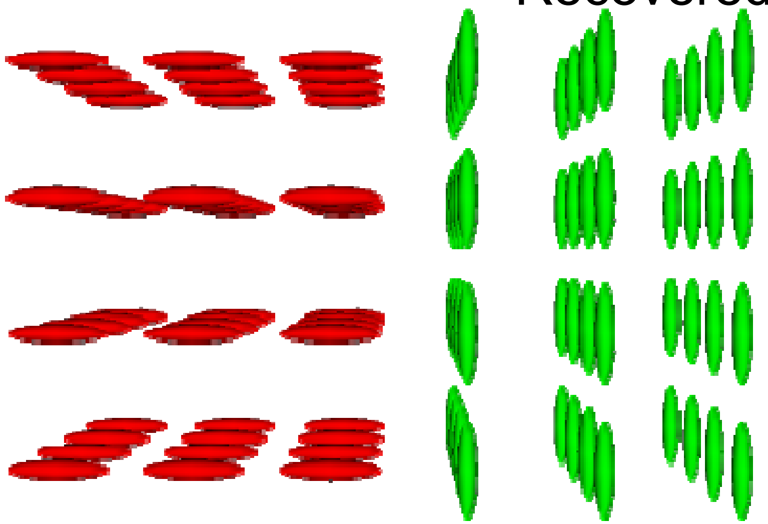
Anisotropic regularization

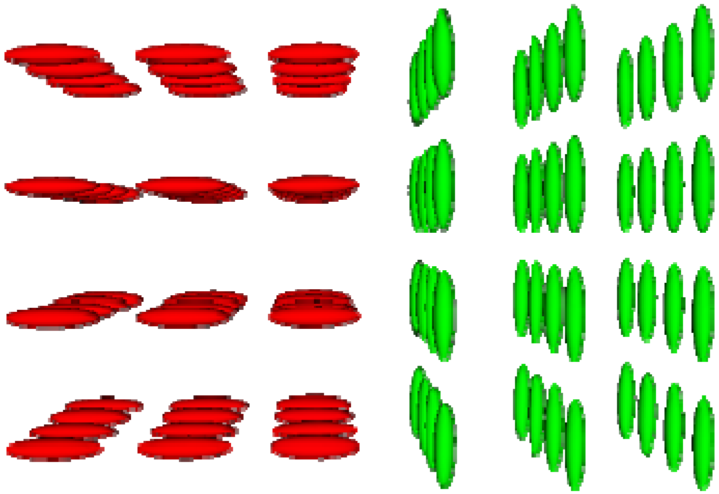
- Perona-Malik 90 / Gerig 92
- Phi functions formalism

Anisotropic filtering

Noisy

Recovered

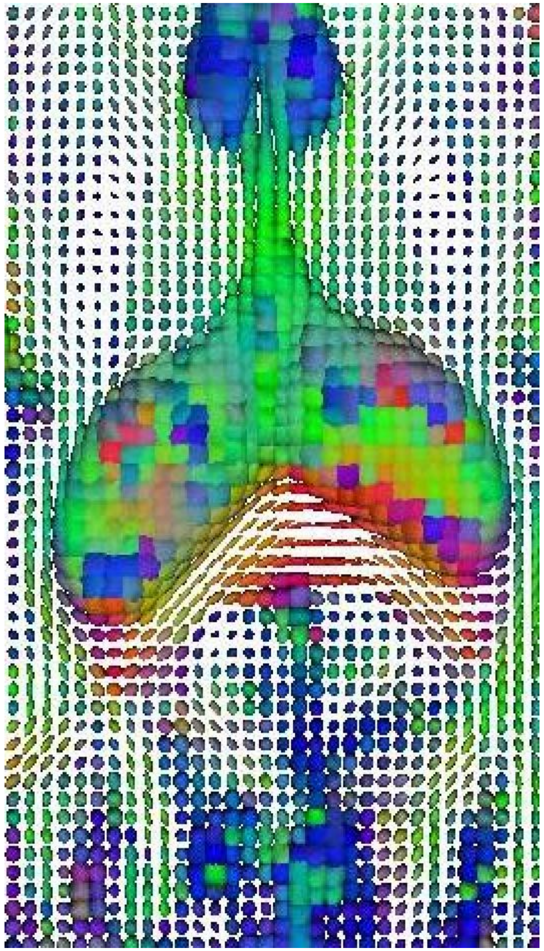




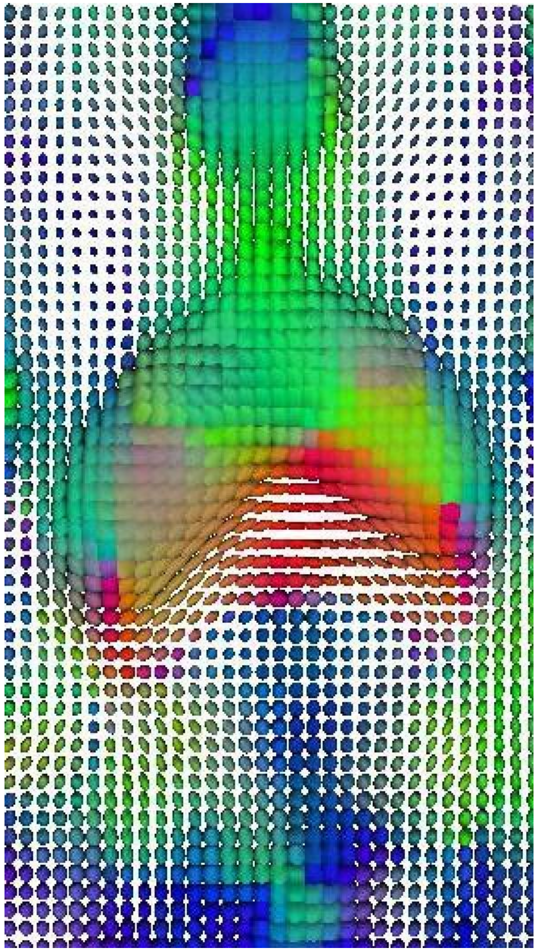
Initial

Anisotropic filtering

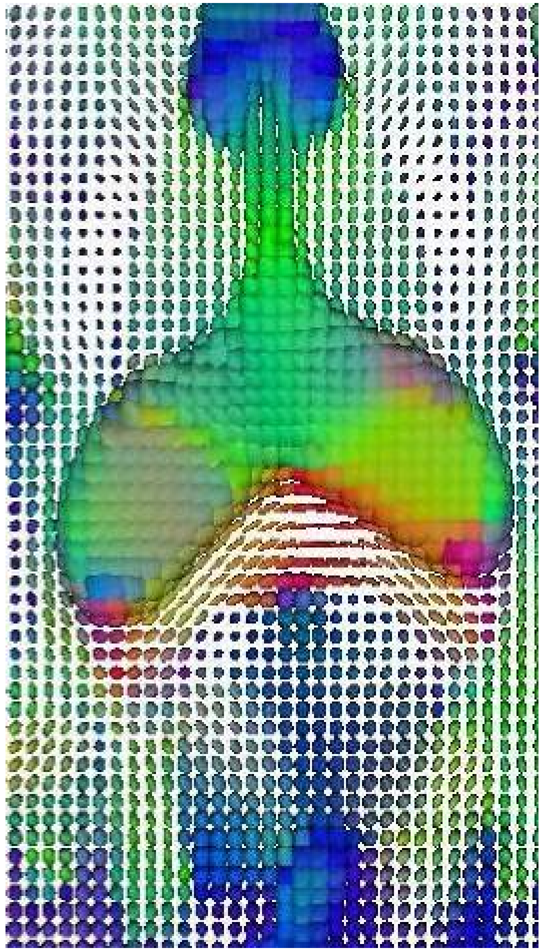
$$\Delta_w \Sigma(x) = \sum_u w\left(\partial_u \Sigma(x)\right)\ \Delta_u \Sigma(x) \quad \text{with} \quad w(t) = \exp(-t^2 / \kappa^2)$$



Raw



Riemann Gaussian



Riemann anisotropic

Log Euclidean Metric on Tensors

Exp/Log: global diffeomorphism Tensors/sym. matrices

- Vector space structure carried from the tangent space to the manifold
  - Log. product $$\Sigma_1 \otimes \Sigma_2 \equiv \exp(\log(\Sigma_1) + \log(\Sigma_2))$$
  - Log scalar product $$\alpha \bullet \Sigma \equiv \exp(\alpha \log(\Sigma)) = \Sigma^{\alpha}$$
  - Bi-invariant metric $$dist(\Sigma_1, \Sigma_2)^2 \equiv \|\log(\Sigma_1) - \log(\Sigma_2)\|^2$$

Properties

- Invariance by the action of similarity transformations only
- Very simple algorithmic framework

[Arsigny, Fillard, Pennec, Ayache, MICCAI 2005, T1, p.115-122]

Log Euclidean vs Affine invariant

- Means are geometric (vs arithmetic for Euclidean)
- Log Euclidean slightly more anisotropic
- Speedup ratio: 7 (aniso. filtering) to >50 (interp.)

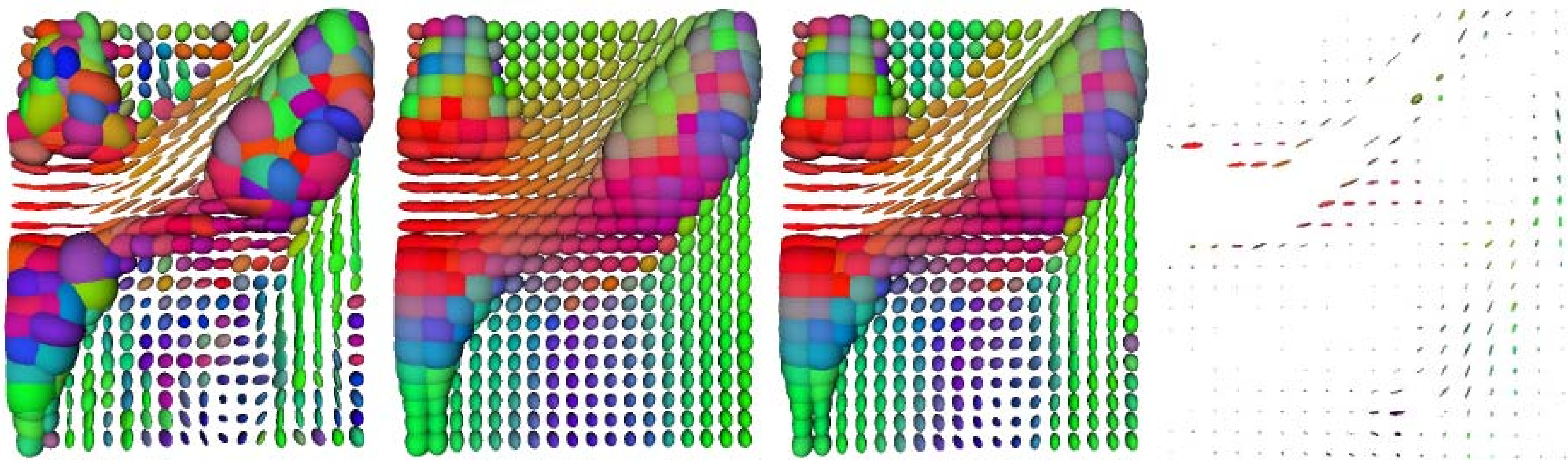
Euclidean

Affine invariant / Log-Euclidean

Log Euclidean vs Affine invariant

Real DTI images: anisotropic filtering

- Difference is not significant
- Speedup of a factor 7 for Log-Euclidean



Original Euclidean Log-Euclidean Diff. LE/affine (x100)

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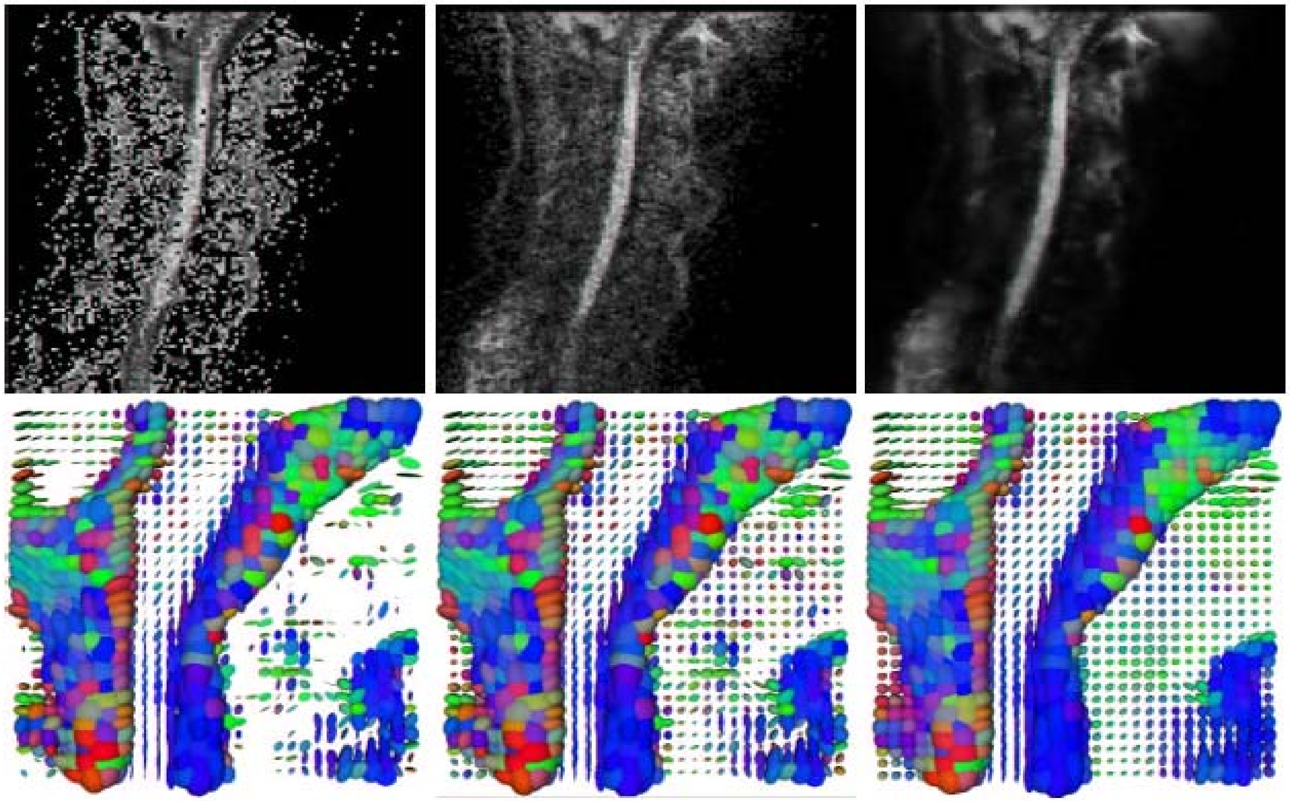
Joint Estimation and regularization from DWI

$$C(\Sigma) = \int \sum_i \left(S_i - S_0 \exp\left(-b\, \mathbf{g}_i^T \Sigma(x)\, \mathbf{g}_i \right) \right)^2 + \Phi\left(\left\| \nabla \Sigma(x) \right\|_{\Sigma(x)}^2 \right)$$

Clinical DTI of the spinal cord

FA

Estimated tensors

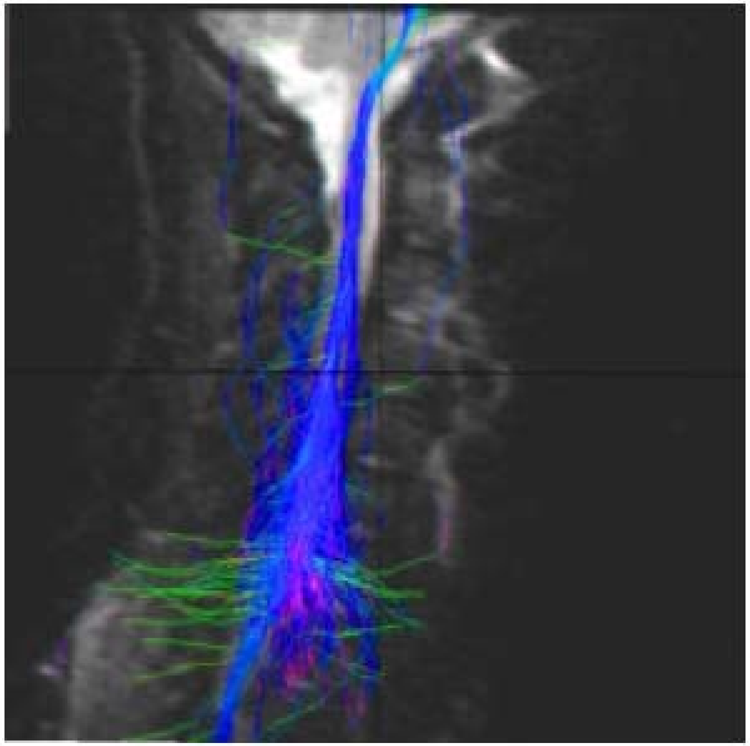


Standard ML Rician MAP Rician

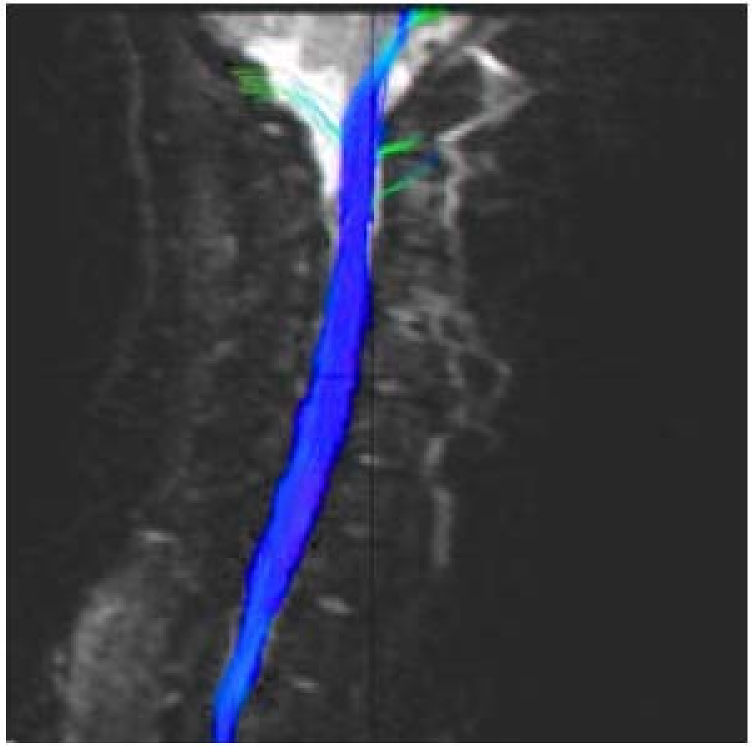
[Fillard, Arsigny, Pennec, Ayache, RR-5607, June 2005]

Joint Estimation and regularization from DWI

Clinical DTI of the spinal cord: fiber tracking

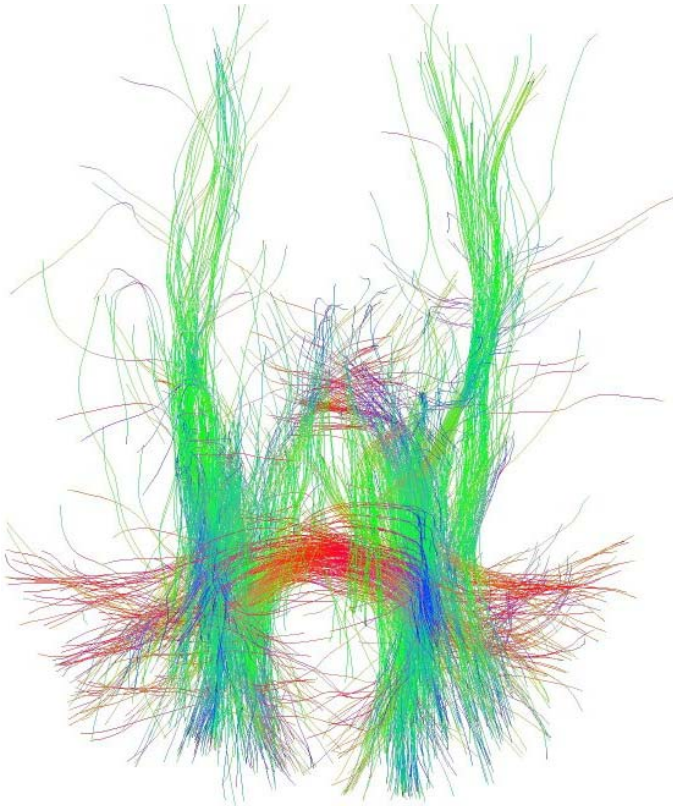


Standard

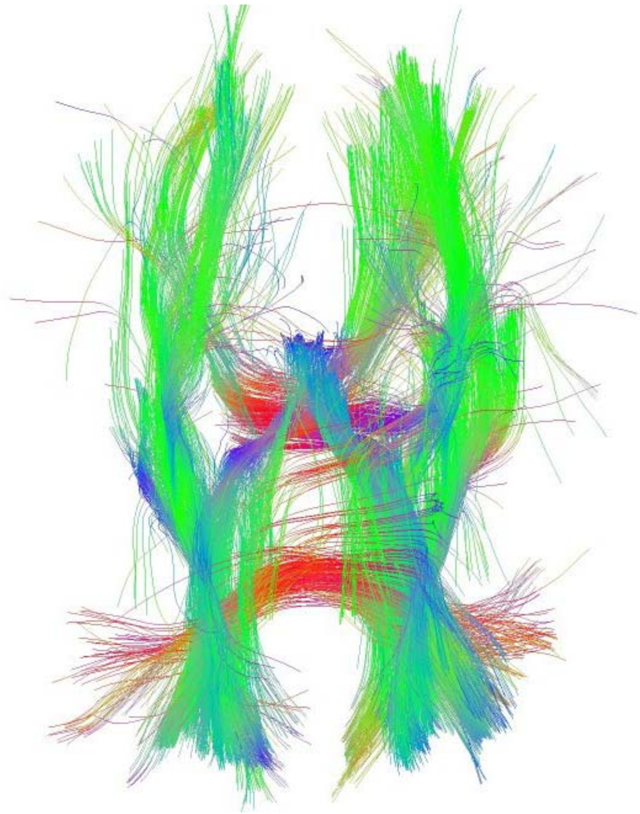


MAP Rician

Impact on fibers tracking



Euclidean interpolation



Riemannian interpolation + anisotropic filtering

From images to anatomy

- Classify fibers into tracts (anatomo-functional architecture)?
- Compare fiber tracts between subjects?

Towards a Statistical Atlas of Cardiac Fiber Structure

[J.M. Peyrat, M. Sermesant, H. Delingette, X. Pennec, C. Xu, E. McVeigh, N. Ayache, INRIA RR , 2006, submitted to MICCAI'06]

Database

- 7 canine hearts from JHU
- Anatomical MRI and DTI

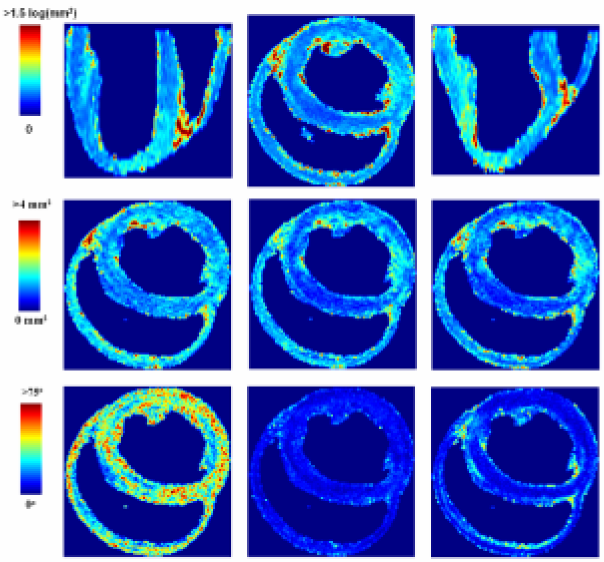
Method

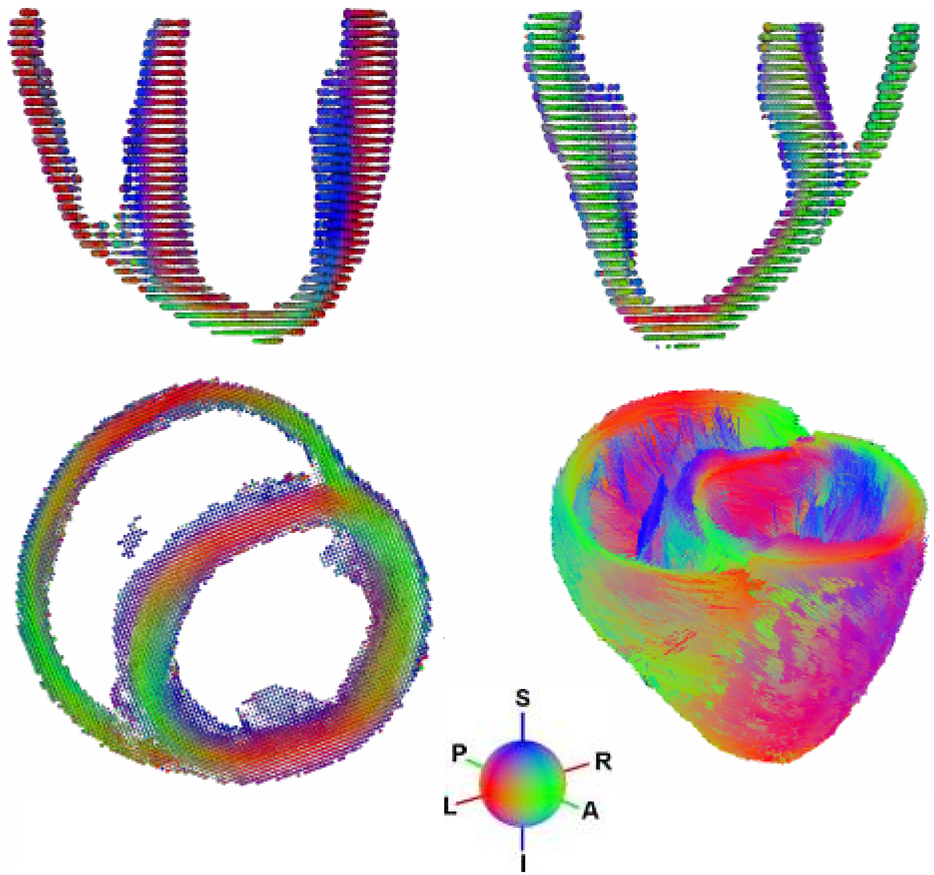
- Normalization based on aMRIs
- Log-Euclidean statistics of Tensors

Norm covariance

Eigenvalues covariance (1^{st}, 2^{nd}, 3^{rd})

Eigenvectors orientation covariance (around 1st, 2nd, 3rd)





Computing on manifolds: a summary

The Riemannian metric easily gives

- Intrinsic measure and probability density functions
- Expectation of a function from M into R (variance, entropy)

Integral or sum in M: minimize an intrinsic functional

- Fréchet / Karcher mean: minimize the variance
- Filtering, convolution: weighted means
- Gaussian distribution: maximize the conditional entropy

The exponential chart corrects for the curvature at the reference point

- Gradient descent: geodesic walking
- Covariance and higher order moments
- Laplace Beltrami for free

Which metric for which problem?

[Pennec, NSIP'99, JMIV 2006, Pennec et al, IJCV 66(1) 2006]

Overview

✓ **The geometric computational framework**

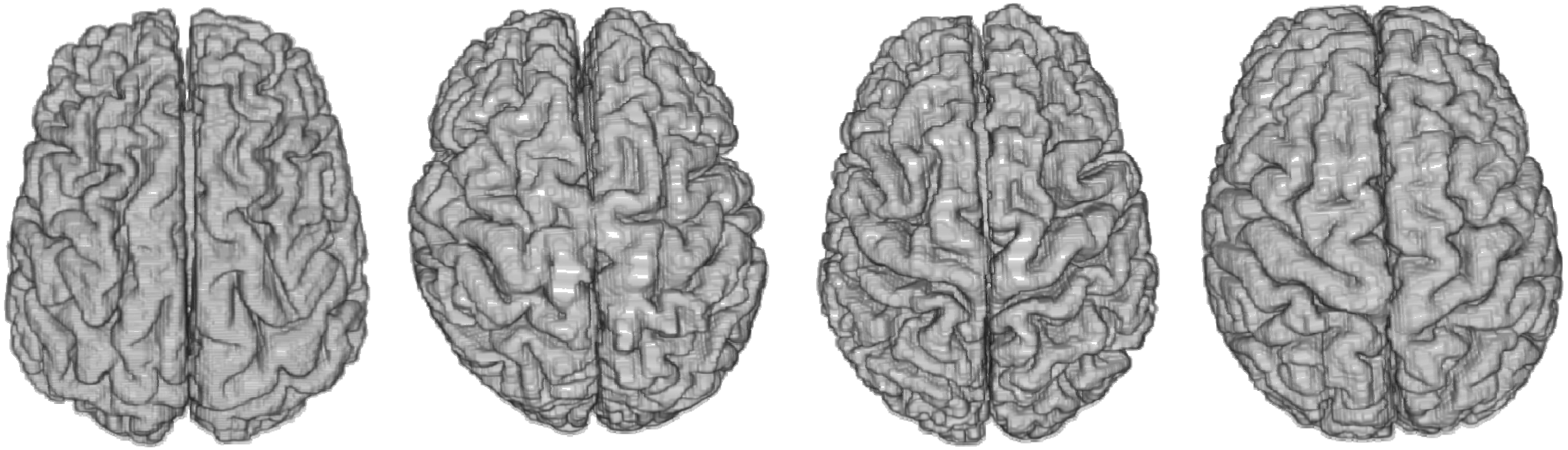
✓ **Statistical tools on pointwise features**

✓ **Manifold-valued images: Tensor Computing**

Metric choices for Computational Neuroanatomy

- ⇨ Morphometry of sulcal lines on the brain
- ◻ Statistics of deformations for non-linear registration

Structural variability of the Cortex



Hierarchy of anatomical manifolds

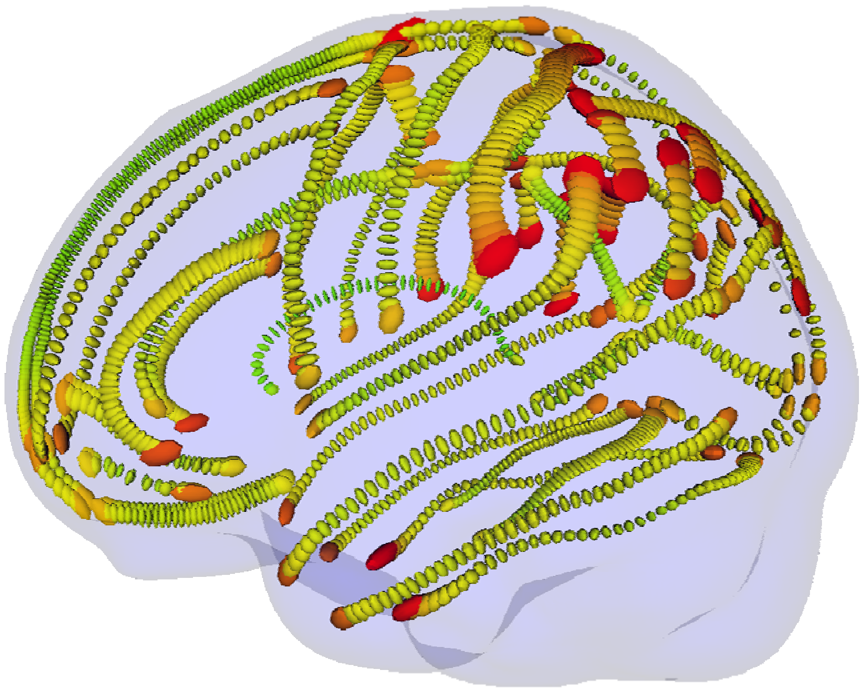
- Landmarks [0D]: AC, PC (Talairach)
- Curves [1D]: crest lines, sulcal lines
- Surfaces [2D]: cortex, sulcal ribbons
- Images [3D functions]: VBM
- Transformations: rigid, multi-affine, diffeomorphisms [TBM]

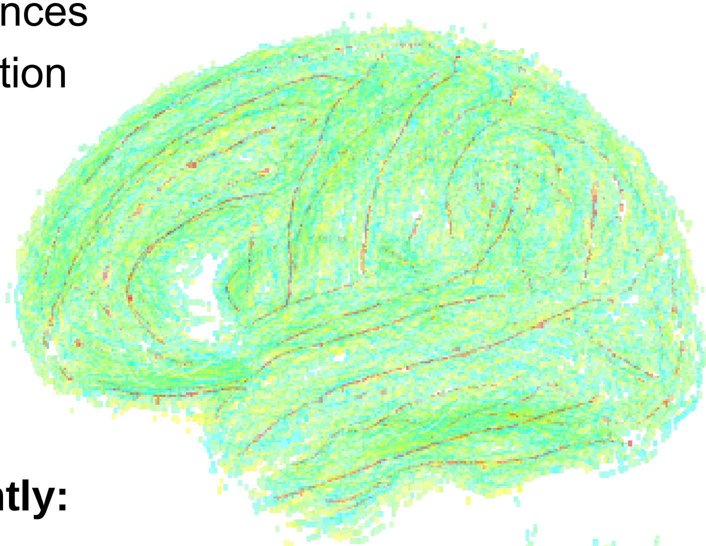
Morphometry of the Cortex from Sucal Lines

Computation of the mean sulci: Alternate minimization of global variance

- Dynamic programming to match the mean to instances
- Gradient descent to compute the mean curve position

Extraction of the covariance tensors

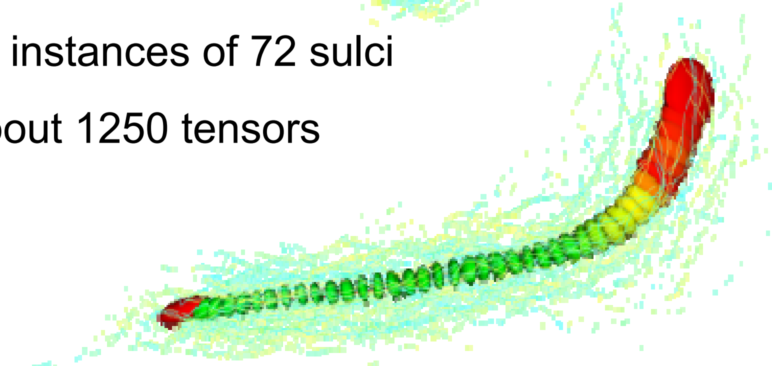




Currently:

80 instances of 72 sulci

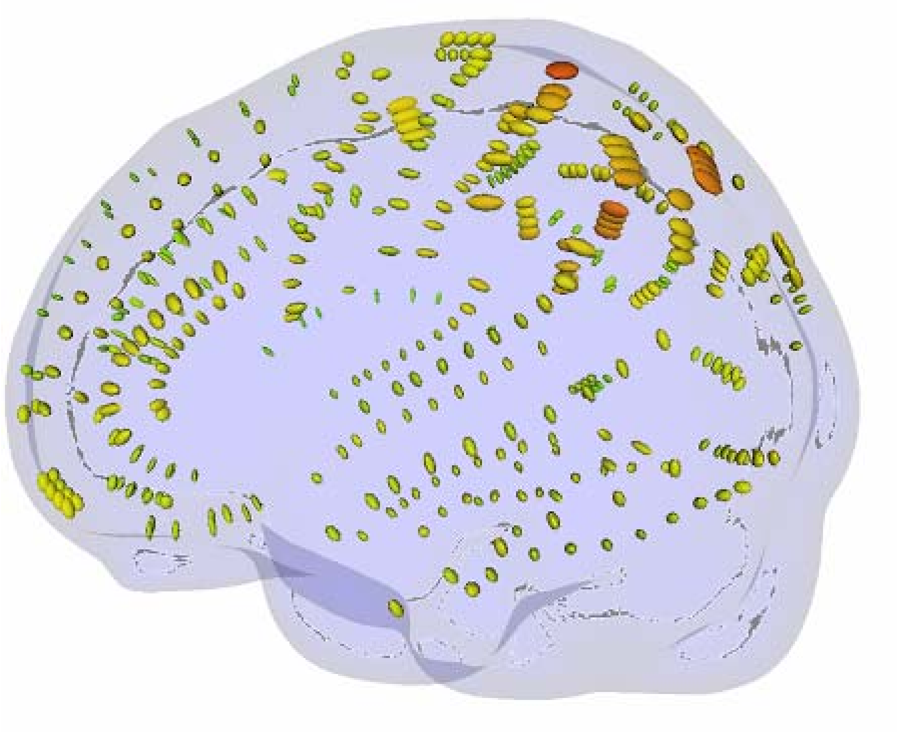
About 1250 tensors



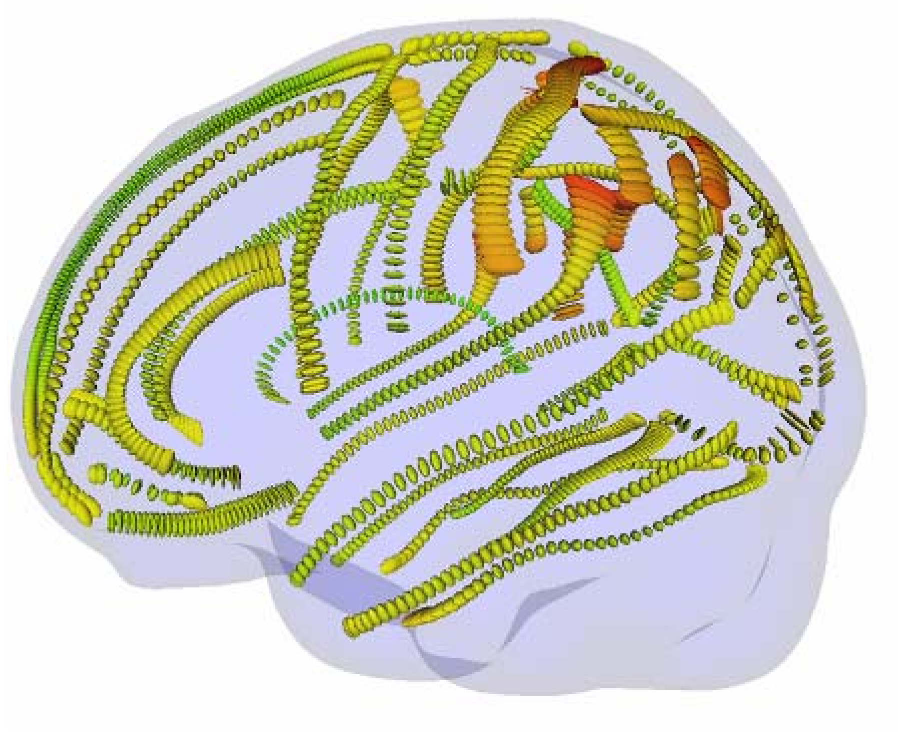
Covariance Tensors along Sylvius Fissure

Collaborative work between Asclepios (INRIA) V. Arsigny, N. Ayache, P. Fillard, X. Pennec and LONI (UCLA) P. Thompson [Fillard et al IPMI05, LNCS 3565:27-38]

Compressed Tensor Representation



Representative Tensors (250)



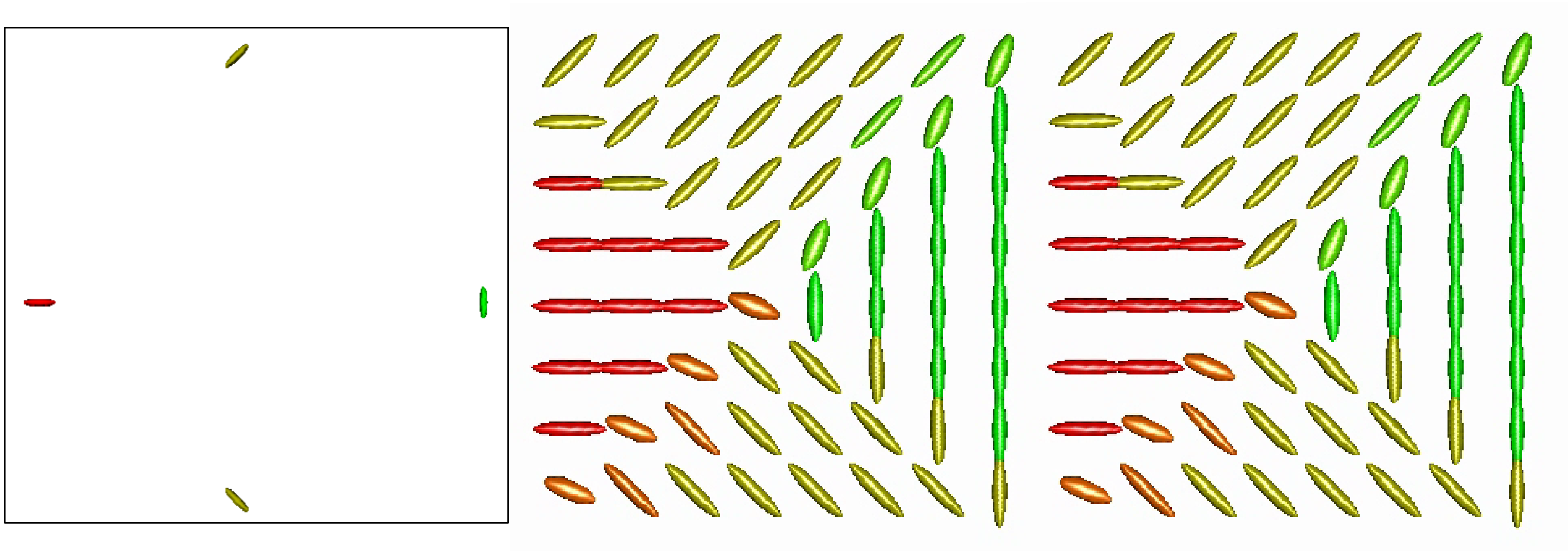
Reconstructed Tensors (1250)
(Riemannian Interpolation)

Extrapolation by Diffusion

$$C(\Sigma) = \frac{1}{2}\int_\Omega \sum_{i=1}^{n} G_\sigma(x - x_i)\, dist(\Sigma(x), \Sigma_i)^2\, dx + \frac{\lambda}{2}\int_\Omega \left\| \nabla \Sigma(x) \right\|_{\Sigma(x)}^2$$

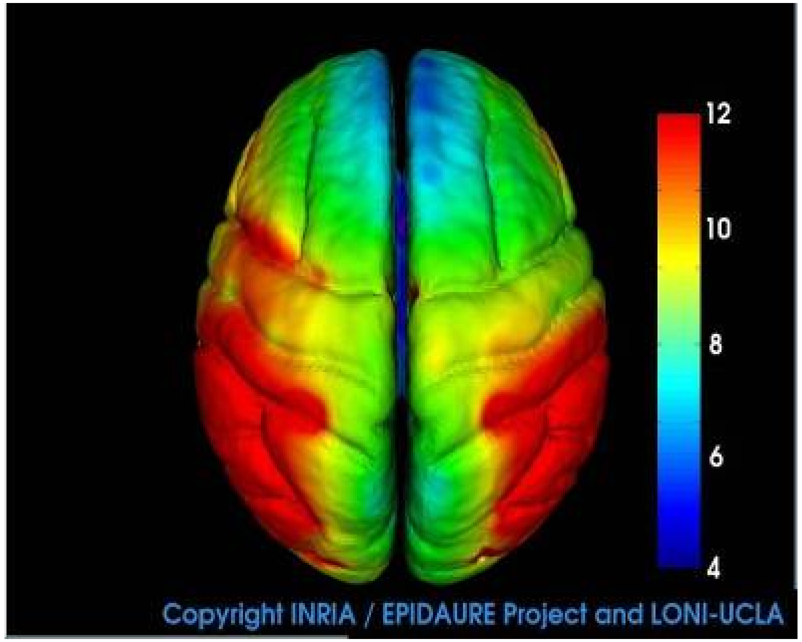
$$\nabla C(\Sigma)(x) = -\sum_{i=1}^{n} G_\sigma(x - x_i)\, \overrightarrow{\Sigma(x)\Sigma_i} - \lambda (\Delta \Sigma)(x)$$

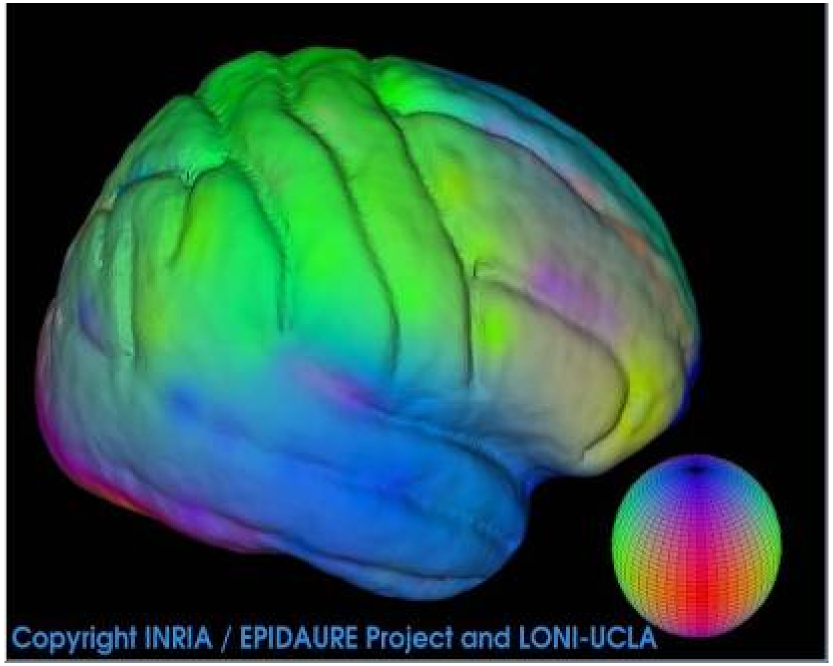
$$\Sigma_{t+1}(x) = \exp_{\Sigma_t(x)}\left(-\varepsilon \nabla C(\Sigma)(x) \right)$$

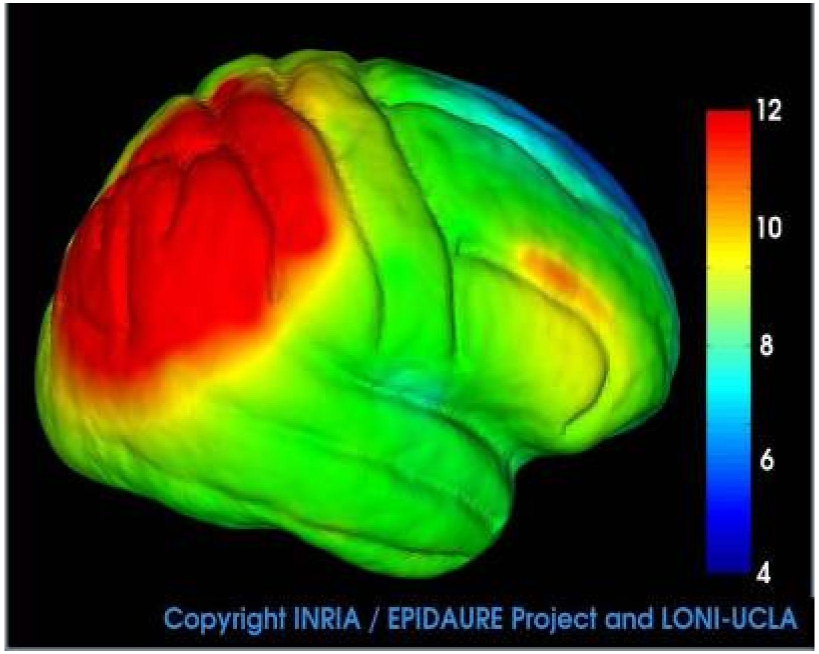


Original tensors | Diffusion λ=0.01 | Diffusion λ=∞

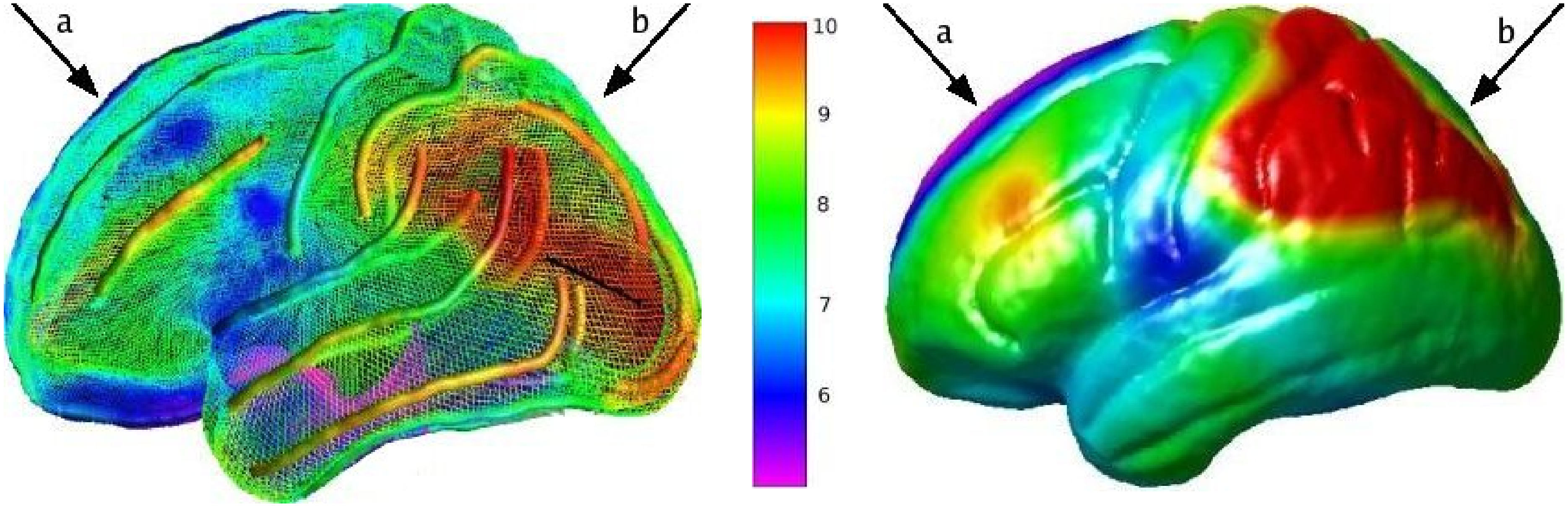
Full Brain extrapolation of the variability







Comparison with cortical surface variability



P. Thompson at al, HMIP, 2000

Average of 15 normal controls by non-linear registration of surfaces

P. Fillard et al, IPMI 05

Extrapolation of our model estimated from 98 subjects with 72 sulci.

Consistent low variability in phylogenetical older areas

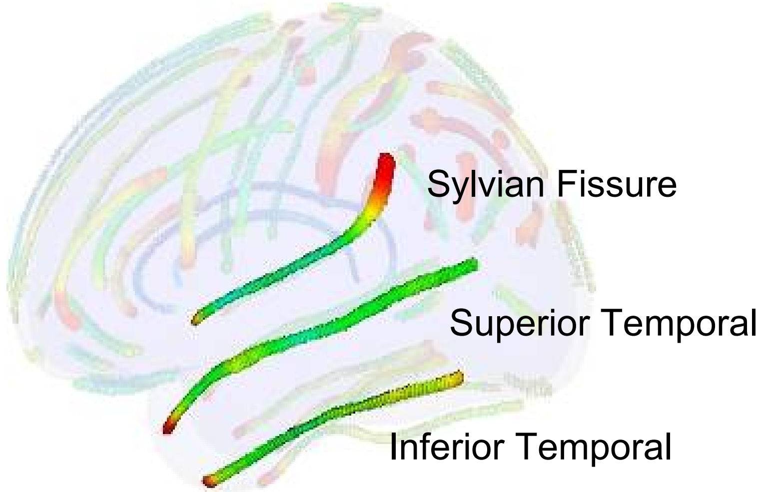
- (a) superior frontal gyrus

Consistent high variability in highly specialized and lateralized areas

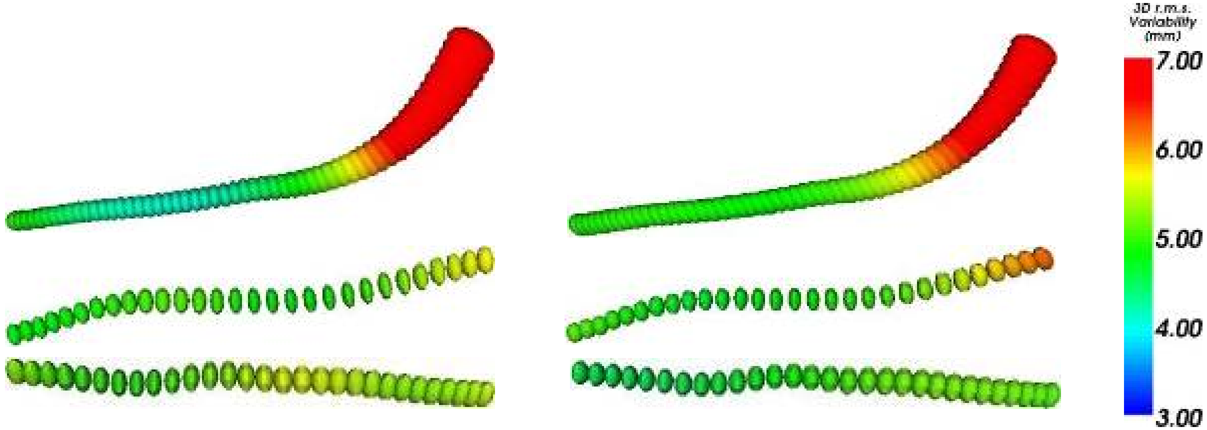
- (b) temporo-parietal cortex

Quantitative Evaluation: Leave One Sulcus Out

- Remove data from one sulcus
- Reconstruct from extrapolation of others

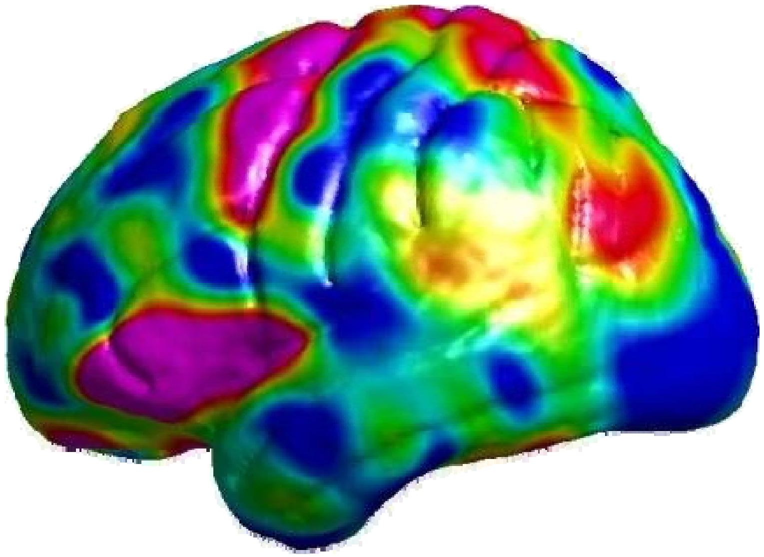


Original tensors **Leave one out reconstructions**

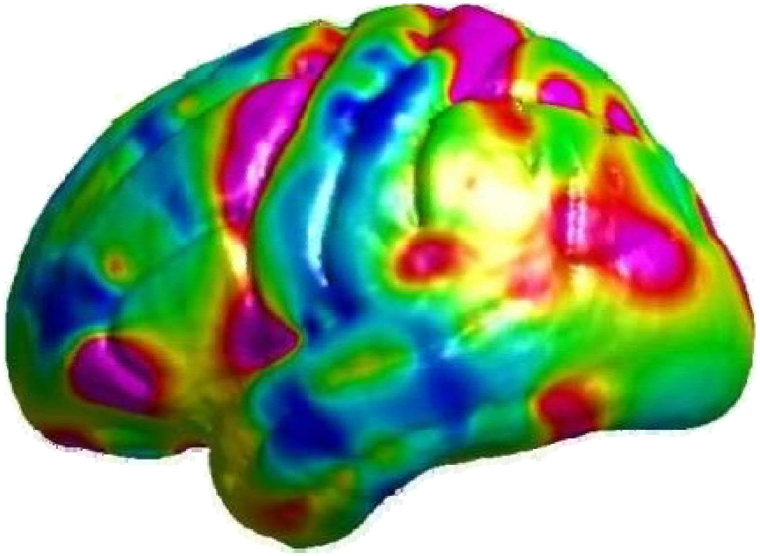


Asymmetry Measures

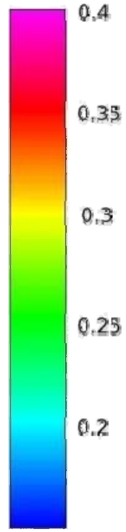
$$dist(\Sigma,\Sigma')^2 = \left\langle \overrightarrow{\Sigma\Sigma'} \,\middle|\, \overrightarrow{\Sigma\Sigma'} \right\rangle_{\Sigma} = \left\| \log(\Sigma^{-1/2}.\Sigma'.\Sigma^{-1/2}) \right\|_{L_2}^2$$



w.r.t the mid-sagittal plane.



w.r.t opposite (left-right) sulci



Greatest asymmetry	Lowest asymmetry
Broca's speech area and Wernicke's language comprehension area	Primary sensorimotor areas

Overview

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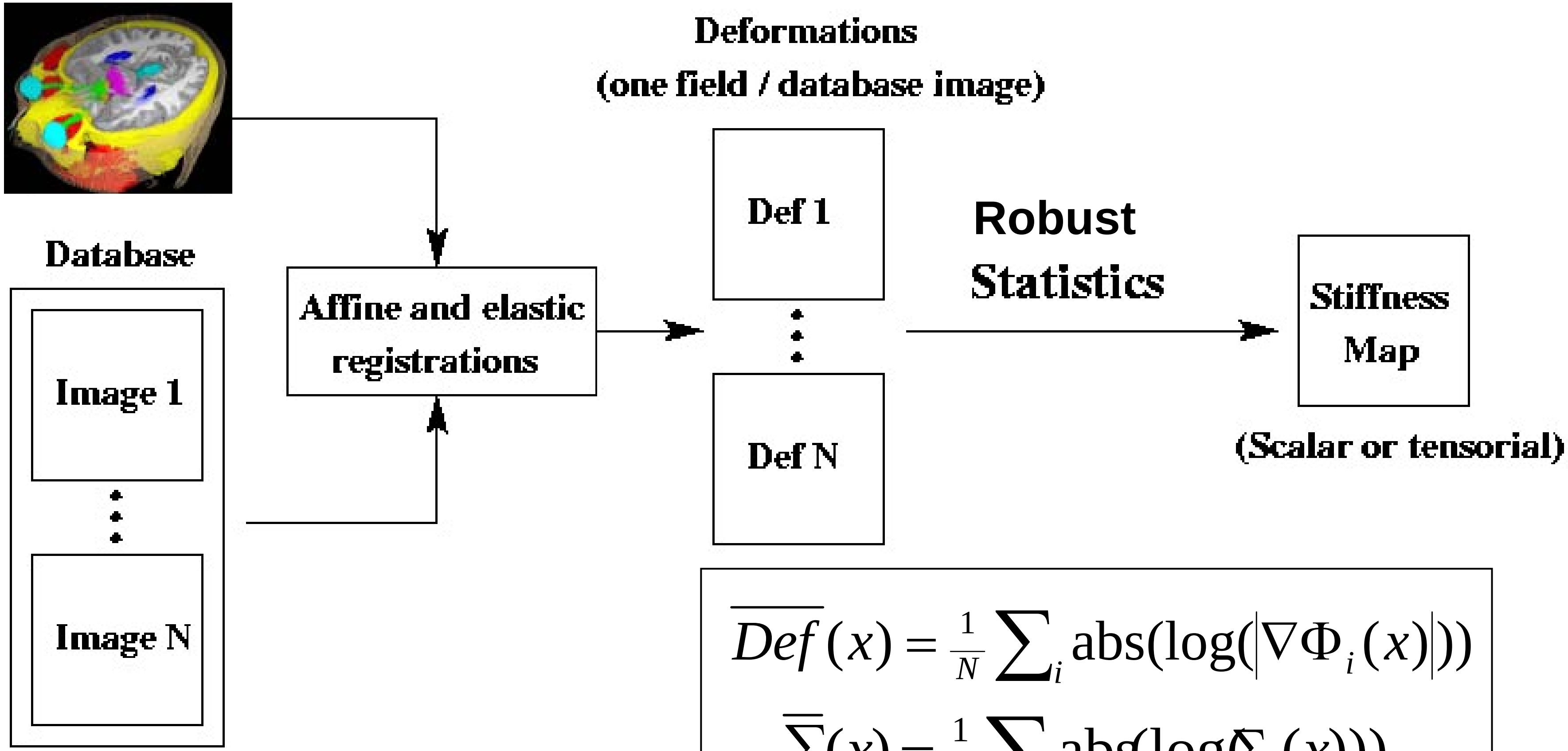
✓ **Manifold-valued images: Tensor Computing**

Metric choices for Computational Neuroanatomy

- ✓ Morphometry of sulcal lines on the brain
- ⇨ Statistics of deformations for non-linear registration

Statistics on the deformation field

- Objective: planning of conformal brain radiotherapy
- 30 patients, 2 to 5 time points (P-Y Bondiau, MD, CAL, Nice)



$$\overline{Def}(x) = \frac{1}{N}\sum_i \mathrm{abs}(\log(|\nabla\Phi_i(x)|))$$

$$\overline{\Sigma}(x) = \frac{1}{N}\sum_i \mathrm{abs}(\log(\Sigma_i(x)))$$

[Commowick, et al, MICCAI 2005, T2, p. 927-931]

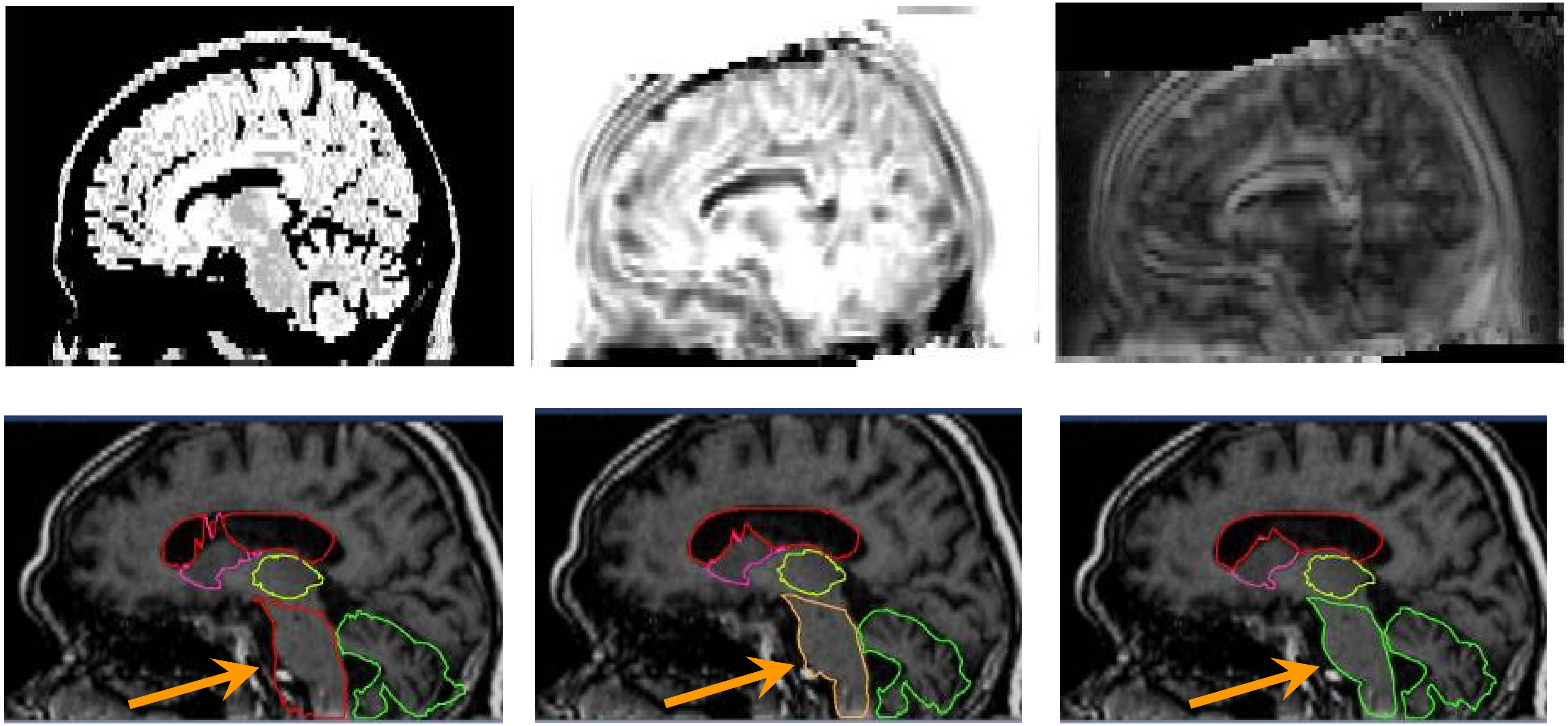
Introducing deformation statistics into RUNA

RUNA [R. Stefanescu et al, Med. Image Analysis 8(3), 2004]

- non linear-registration with non-stationary regularization
- Scalar or tensor stiffness map

$$D(x) = (Id + \lambda \overline{\sum}(x))^{-1}$$

Heuristic RUNA stiffness | **Scalar statistical stiffness** | **Tensor stat. stiffness (FA)**



Riemannian elasticity for Non-linear elastic regularization

[Pennec, et al, MICCAI 2005, LNCS 3750:943-950]

Gradient descent $C(\Phi) = \mathrm{Sim}(\mathrm{Images}, \Phi) + \mathrm{Reg}(\Phi)$ $\Phi_{t+1} = \Phi_t - \kappa \nabla C(\Phi_t)$

Regularization

- Local deformation measure: Cauchy Green strain tensor $\Sigma = \nabla\Phi^t.\nabla\Phi$
  Id for local rotations, small for local contractions, Large for local expansions
- St Venant Kirchoff elastic energy $\mathrm{Reg}(\Phi) = \int \mu\,\mathrm{Tr}\left((\Sigma - I)^2\right) + \frac{\lambda}{2}\mathrm{Tr}(\Sigma - I)^2$

Problems

- Elasticity is not symmetric $\mathrm{d}(\Sigma,0) = \mathrm{d}(\Sigma,2\Sigma)$
- Statistics are not easy to include

Idea: Replace the Euclidean by the Log-Euclidean metric $\mathrm{Tr}\left((\Sigma - I)^2\right) \rightarrow \mathrm{dist}_{LE}(\Sigma, I)^2 = \|\log(\Sigma)\|^2$

Statistics on strain tensors

- Mean, covariance, Mahalanobis computed in Log-space
  $$\mathrm{Re}\,g(\Phi) = \int \mathrm{Vect}\left(\log(\Sigma) - \overline{W}\right)^T.\mathrm{Cov}^{-1}.\mathrm{Vect}\left(\log(\Sigma) - \overline{W}\right)$$
- Isotropic Riemannian Elasticity $\mathrm{Reg}_{\mathrm{iso}}(\Phi) = \int \mu\,\mathrm{Tr}\left(\log(\Sigma)^2\right) + \frac{\lambda}{2}\mathrm{Tr}(\log(\Sigma))^2$

Conclusion : geometry and statistics

A Statistical computing framework on Riemannian manifolds

- Mean, Covariance, statistical tests…
- Interpolation, diffusion, filtering…
- Which metric for which problem?

Important applications in Medical Imaging

- Medical Image Analysis
  - Evaluation of registration performances
  - Diffusion tensor imaging
- Building models of living systems (spine, brain, heart…)

Noise models for real anatomical data

- Physically grounded noise models for measurements
- Anatomically acceptable families of deformation metrics
- Spatial correlation between neighbors… and distant points
- … and statistics to measure and validate that!

Challenges of Computational Anatomy

Computing on manifolds

- Parametric families of metrics (models of the Green's function)
- Topological changes
- Evolution: growth, pathologies

Build models from multiple sources

- Curves, surfaces [cortex, sulcal ribbons]
- Volume variability [Voxel Based Morphometry, Riemannian elasticity]
- Diffusion tensor imaging [fibers, tracts, atlas]

Compare and combine statistics on anatomical manifolds

- Compare information from landmarks, courves, surfaces
- Validate methods and models by consensus
- Integrative model (transformations ?)

Couple modeling and statistical learning

- Statistical estimation of model's parameters (anatomical + physiological)
- Use models as a prior for inter-subject registration / segmentation
- Need large database and distributed processing/algorithms (GRIDS)

MFCA-2006: International Workshop on Mathematical Foundations of Computational Anatomy

Geometrical and Statistical Methods for Modelling Biological Shape Variability

October 1st, Copenhagen, in conjunction with MICCAI'06

Goal is to foster interactions between geometry and statistics in non-linear image and surface registration in the context of computational anatomy with a special emphasis on theoretical developments.

Chairs: Xavier Pennec (Asclepios, INRIA), Sarang Joshi (SCI, Univ Utah, USA)

- Riemannian and group theoretical methods on non-linear transformation spaces
- Advanced statistics on deformations and shapes
- Metrics for computational anatomy
- Geometry and statistics of surfaces

www.miccai2006.dk –> Workshops -> MFCA06

www-sop.inria.fr/asclepios/events/MFCA06/

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[Papers available at http://www-sop.inria.fr/asclepios/Biblio]

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