

Geometrical Analysis of Facial Surfaces

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joint work with:

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MIA 2006

1 Motivation.

2 Facial surface matching using planar facial curves.

- Facial surface representation.
 - Geodesic path between facial curves.
- Comparing facial surfaces.
- Some experimental results.
 - Limitation and extension of this method.

3 Geometrical analysis of facial surfaces.

- A brief summary of 3D curves analysis.
- Facial surface representation.
- Geodesic path between facial surfaces.
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Motivation: Study Shapes of Surfaces

- We want to compute and utilize statistics of facial surfaces.
- How should we represent a facial surface?
- Given two facial surfaces, we want to compute a geodesic path on the space of facial surfaces.
- We want to evaluate the dissimilarity between two facial surfaces.



Facial surface matching using planar facial curves

C. Samir, A. Srivastava, and M. Daoudi IEEE PAMI Nov 2006, ICASSP 2006

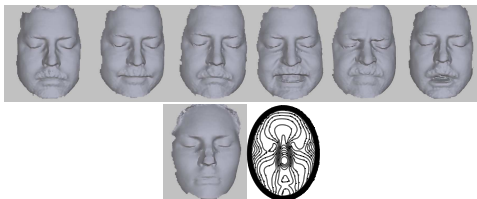
- **Goal:** Analysis of facial surfaces (no canonical parametrization).



- **Approach:** Approximate a surface using a collection of closed curves, and then compare surfaces by comparing their corresponding curves.

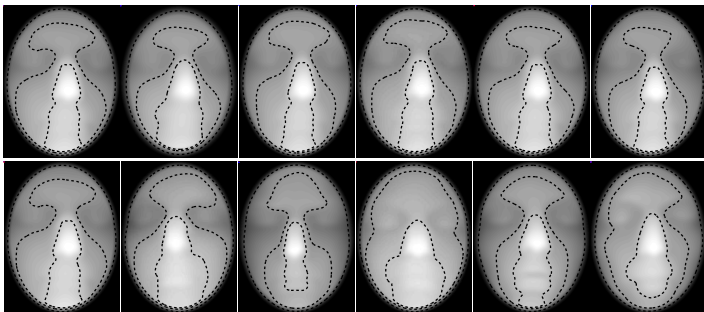


Facial Curves



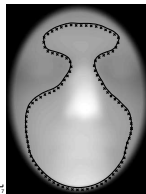
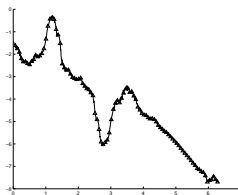
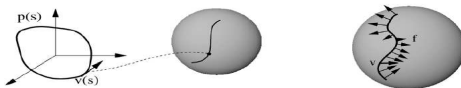
- Let S be a facial surface denoting a scanned face.
- Let $F : S \mapsto \mathbb{R}$ be a continuous function, **depth function here**, and let C_λ denote the level curve of F , called a **facial curve**, for the value $\lambda \in F(S)$, i.e. $C_\lambda = \{p \in S | F(p) = \lambda\} \subset S$. We can reconstruct S through these level curves according to $S = \cup_\lambda C_\lambda$.

Examples of Extracted Facial Curves



Range images: Top same person under different expressions. Bottom different persons under same expression.

- Coordinate function $\alpha(s) \in \mathbb{R}^2$ of C_λ relates to the direction function $\theta(s)$ according to $\dot{\alpha}(s) = e^{j\theta(s)}$, $j = \sqrt{-1}$, when s is the arc-length parameter.



E. Klassen, A. Srivastava, and X. Mio IEEE PAMI 2004

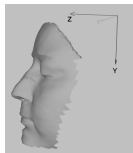
- For a closed curve, θ must satisfy the *closure condition*:

$$\int_0^{2\pi} \exp(j \theta(s)) ds = 0.$$
- To make shapes invariant to planar rotation, restrict to angle functions such that, $\frac{1}{2\pi} \int_0^{2\pi} \theta(s) ds = \pi.$
- $\mathcal{C} = \{\theta \mid \frac{1}{2\pi} \int_0^{2\pi} \theta(s) ds = \pi, \int_0^{2\pi} e^{j\theta(s)} ds = 0\}.$
- To remove the re-parametrization group \mathbb{S}^1 (relating to different placements of origin, point with $s = 0$, on the same curve), define the quotient space $\mathcal{D} \equiv \mathcal{C}/\mathbb{S}^1$ as the shape space.
- \mathcal{D} is a Riemannian manifold.

Are facial curves invariant to transformations ?

Our goal is to analyze shape of S invariant to its rigid rotations and translations, and uniform scalings.

- Our technique for comparing shapes of closed curves will be automatically invariant to planar transformations in $\text{SO}(2) \times \mathbb{R}^2$ and the z -translations in \mathbb{R} .
- The variability due to changes in z direction (or gaze direction) in \mathbb{S}^2 is not removed automatically. Two solution are then proposed:
- Acquisition Control

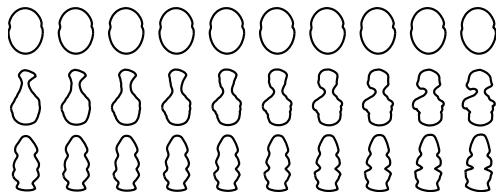


- Alignment algorithm(s): ...

Geodesic Path between facial curves

Different steps to compare two facial curves:

- Riemannian metric $\langle g_1, g_2 \rangle = \int_0^{2\pi} g_1(s)g_2(s)ds$.
- **Minimum energy to bend** one curve into the other.
- Geodesic path between two facial curves.
- Distance d_c (the length of the geodesic) between two facial curves.



Distance between facial surfaces

- Let's $\{C_\lambda^1 | \lambda \in \Lambda\}$ and $\{C_\lambda^2 | \lambda \in \Lambda\}$ be the collections of facial curves associated with the two surfaces,
- Let d_c denotes the distance between two facial curves, associated to their geodesic path. Two possible distances between them are defined: assuming $|\Lambda|$ to be finite,

$$d_e(S^1, S^2) = \left(\sum_{\lambda \in \Lambda} d_c(C_\lambda^1, C_\lambda^2)^2 \right)^{1/2}.$$

$$d_g(S^1, S^2) = \left(\prod_{\lambda \in \Lambda} d_c(C_\lambda^1, C_\lambda^2) \right)^{1/|\Lambda|}.$$

Where d_e denotes the Euclidean mean and d_g the Geometric one.

Algorithm

Step 1: Mesh refinement, generation of range images, and masking of the range images.

Step 2: Level curves extractions from range images.

Step 3: Computation of an angle function for each extracted curve, fitting a spline through its graph and re-sampling for arc-length parametrization.

Step 4: Computation of geodesic lengths between respective facial curves, and calculation of a distance between facial surfaces.

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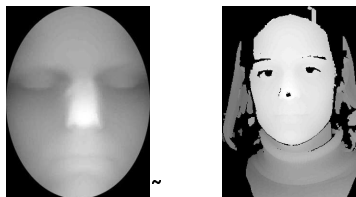
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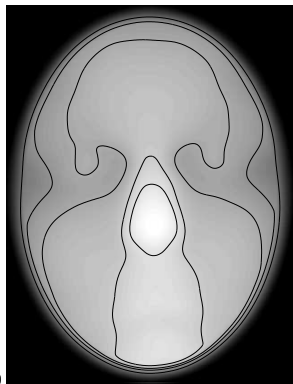
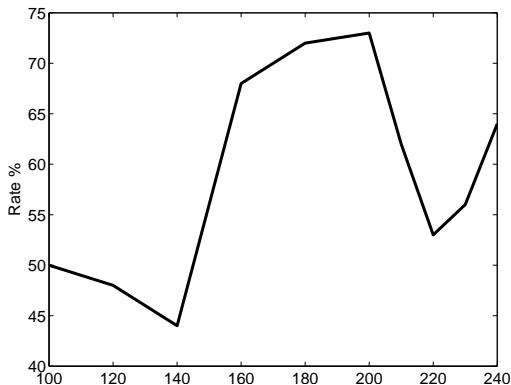
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Preprocessing

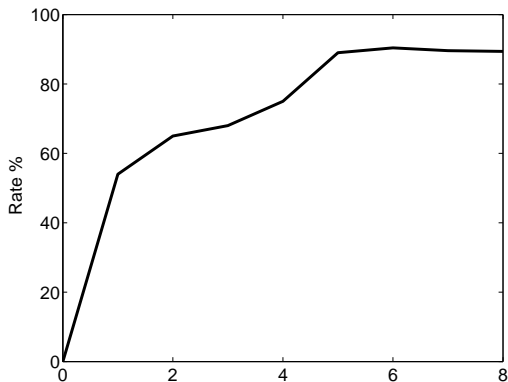
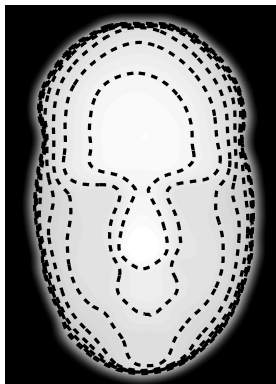
- FSU Database (private): 300 scans, six facial expressions/person. Assigning r ($r = 1, 2, \dots, 5$) faces per person as a gallery set, and the remaining $6 - r$ faces/person as a probe set.
- Notre Dame Database (public). We have divided the remaining 740 scans of 162 subjects into 470 gallery and 270.
- Nearest neighbor recognition.



Pertinent information

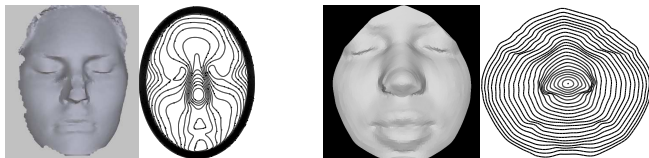


Coarse representation??



The limitation and extension

- The depth function depends on the gaze direction, by serious rotation around X-Axis, the extracted facial curves will largely differ for same person. To resolve this problem and make the method invariant to rigid transformation, we used the geodesic length function either than depth function.

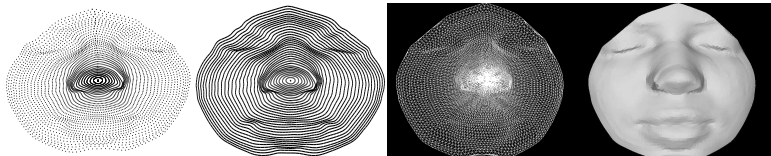


Extracted facial curves using depth function in the left and the geodesic length function in the right.

Geometrical analysis of facial surfaces

Extracted (3D) Facial curves

- The depth function is replaced by the geodesic length function:
 $f(x) = d_g(x, q)$ where q is the reference point (nose tip here), and $d_g(.,.)$ is the shortest distance on a facial surface.
- A surface refinement (good resolution, filling holes...).
- The resulted facial curves (level set) are 3D and are elements of $(C \times \mathbb{R}_+)$ (unit closed curve, length).
- \Rightarrow Need of a 3D curves analysis.

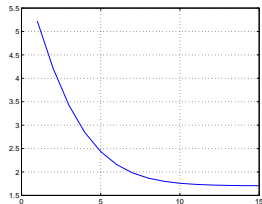
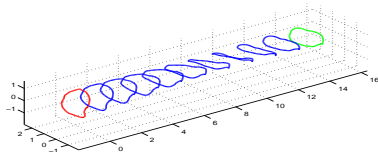


Different representations of S .

A summary of 3D curves Analysis: Klassen et al SIAM in review

- Same representation (angle function) as previous study.
- \mathcal{C} is a differential manifold of 3D closed curves.
- $\alpha : [0, 1] \rightarrow \mathcal{C}$ is a differential path in \mathcal{C} .
- Given two curves C_0 and C_1 , and a path α such that $\alpha(0) = C_0$ and $\alpha(1) = C_1$, they used a path-straightening method on α to find the geodesic between them in \mathcal{C} , noted ϕ here.
- The length of ϕ is a distance, and will be noted d_c .

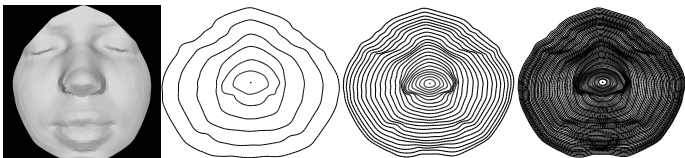
Comparing Shapes of 3D Facial Curves



The geodesic path between two facial curves and the evolution of the energy.

Representations of facial surfaces

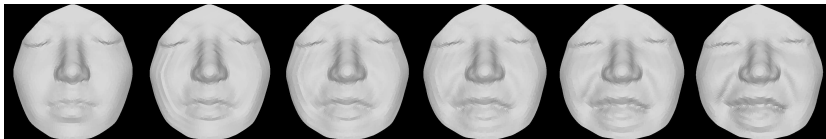
- Let α be a mapping, such $\alpha: [0, 1] \rightarrow (C \times \mathbb{R}_+)$,
 - C is the space of closed curves in \mathbb{R}^3 .
 - α can be considered as an indexed path in C .
- A facial surface is represented as a path in the space of closed curves.



S represented by N finite number of facial curves ($N = 7, 24, 70$).

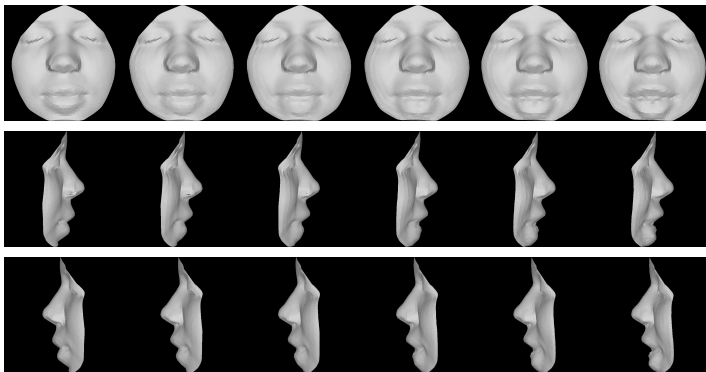
Geodesics Between Facial Surfaces

Let S_0 and S_1 be any two given facial surfaces, and α_0 and α_1 be the corresponding elements in \mathcal{H} , respectively. Our goal is to construct a geodesic path $\Psi(t)$ in \mathcal{H} , parameterized by time t , such $\Psi(0) = \alpha_0$ and $\Psi(1) = \alpha_1$. A distance between α_0 and α_1 is the length of the corresponding geodesic.



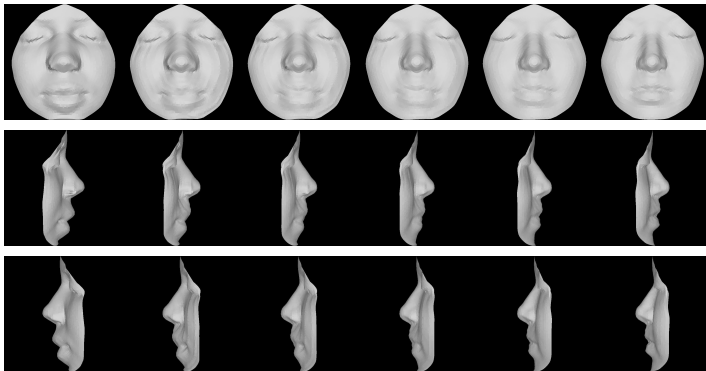
Geodesic Paths between the same person with two different facial expressions .

Same person



Geodesic Paths between the same person viewed from different viewpoints.

Different persons



Geodesic Paths between two different persons viewed from different viewpoints.

Summary

- Presented a Facial surface matching method using planar facial curves analysis.
 - A new representation of facial surface based on curves.
 - A facial surfaces matching method.
 - Some Experimental results.
- Presented a brief summary of 3D curves analysis
- Presented an approach for computing geodesics paths between facial surfaces.
- Showed examples of paths between two facial surfaces.

Questions:

Thank you for your attention !